MINISTRY OF EDUCATION AND SCIENCE
KYIV NATIONAL UNIVERSITY OF CONSTRUCTION AND ARCHITECTURE

STRENGTH OF MATERIALS

Construction Diagrams of Internal Forces.
Methodological recommendations for performing calculation and graphic work № 2

Methodical recommendations, tasks and

examples for performing

calculation and graphic works

for students pursuing the first (Bachelor's) level of higher education in specialties

G19 Building and civil engineering.

Educational program: Heating, Gas Supply and Ventilation Engineering (HV);

Water Supply and Drainage (WSD)

Kyiv 2026

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The methodological guidelines contain variants of individual assignments and detailed plans for performing calculation and graphic works for the course "Strength of Materials", as well as solution examples for each task. The section "Construction of Internal Force Diagrams" is covered.

For students pursuing the first (Bachelor’s) level of higher education, in specialties **G19 Building and Civil Engineering**. Educational programs: Heat and Gas Supply and Ventilation (HGSV); Water Supply and Wastewater Disposal (WSWD).

Опір матеріалів: Побудова епюр внутрішніх зусиль: методичні вказівки,
О-61завдання та приклади виконання розрахункових і графічних робіт [електронний ресурс]. / укладачі: О.О. Кошевий, М.О. Янсонс – Київ: КНУБА, 2026. – 46 с.

містять варіанти індивідуальних завдань та детальні плани виконання розрахункових і графічних робіт з курсу «Опір матеріалів», а також приклади розв’язання кожного завдання. Розглянуто розділ «Побудова епюр внутрішніх зусиль».

Призначено для здобувачів першого (Бакалаврського) рівня вищої освіти за програмою **G19 «Будівництво та цивільна інженерія»,** що навчаються англійською мовою. Освітні програми: «Теплогазопостачання і вентиляція» (ТГВ); «Водопостачання та водовідведення» (ВВ).

GENERAL PROVISIONS

The methodological guidelines are intended to assist students in completing the calculation and graphic work for the course "Strength of Materials" in the section: "Construction of Internal Force Diagrams." The calculation and graphic work must be completed by the student according to an individual assignment, the conditions of which are determined by the lecturer. The initial data (calculation schemes and numerical values) are selected by the student from the appendices (see Appendices 1, 2, 3) according to their personal code in the form of a three-digit number. Explanations and calculations must be performed on a computer using MS Word. For each assignment, a plan and recommendations for step-by-step calculations are provided.

The calculation and graphic work is prepared on A4 sheets, bound on the left side.

The title page of the work is formatted according to the provided sample.

The calculation and graphic work is prepared on A4 sheets, bound on the left side. The title page of the work is formatted according to the following sample:

Ministry of Education and Science of Ukraine

Kyiv National University of Construction and Architecture

Department of Strength of Materials

Calculation and Graphic Work No.__

"______________________________"

/Topic/

Prepared by:

Student

/Specialty, Year, Group/

____

/Surname, Initials/

Supervisor:

/Surname, Initials/

Kyiv – 20__

Explanations and calculations must be performed on one side of the sheet using a pen, while drawings must be completed in pencil. Computer-aided drafting is permitted.

For each assignment, a plan and recommendations for step-by-step calculations are provided. An example of completing the calculation and graphic work (CGW) is given for illustration. When performing each stage of the calculation, the student must first write down the calculation formulas, substitute the numerical values, and record the result of the computation in the appropriate units of measurement (SI system).

The calculation and graphic work is considered completed only after its defense.

PURPOSE OF THE WORK:

To familiarize students with the basic concepts and the classical method of constructing internal force diagrams; with the main approaches and principles of analysis for normal and shear stresses; to provide practical skills in performing engineering calculations for determining internal forces required for strength, stiffness, and stability analysis; to teach methods for processing and verifying the obtained calculation results.

Construction of Internal Force Diagrams

1.1 General Provisions.

When a body is subjected to arbitrary external loads, a change occurs in the internal forces of interaction between its particles (molecules, atoms, etc.). In Strength of Materials, this change in interaction forces is referred to as **internal forces**. To determine internal forces, the method of sections is used. To do this, the body is cut at a certain point by a plane (usually perpendicular to the body's axis) into two parts—left and right. The equilibrium of one of the cut-off parts is then examined under the action of external loads and the internal forces arising in the cross-sectional plane. Internal forces per unit area of the cross-section are called **stresses**. They can be reduced to the centroid of the cross-section. The projections of the resultant force vector and the resultant moment vector of internal forces onto the longitudinal axis **X** (or, for a curvilinear axis, onto the tangent to this axis) and onto the principal central axes of the cross-section ***Y*** and ***Z*** are called **internal forces and moments** in the given section (Fig. 2.1). In the general case of bar loading, there are six of them.

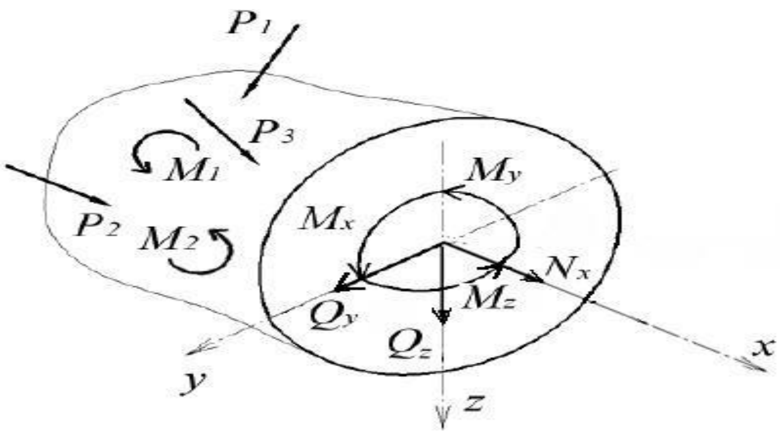


Fig. 2.1

N – normal (axial) force; Q_y, Q_z – shear forces; M_x (M_{kp}) – torque (torsional moment); M_y, M_z – bending moments.

Hereinafter, **external forces** are understood as distributed loads, concentrated forces, and concentrated moments applied to the longitudinal axis of the bar, including support reactions.

The **normal force N** in a cross-section is equal to the algebraic sum of the projections onto the longitudinal axis **X** of the bar (or onto the tangent to it) of all external forces applied to one of the cut-off parts.

The **shear force** $\boldsymbol{Q_y}$ (or Q_z) in a cross-section is equal to the algebraic sum of the projections onto the axis $\boldsymbol{Y}$ (or $\boldsymbol{Z}$), which is perpendicular (normal) to the bar's axis, of all external forces applied to one of the cut-off parts.

The **bending moment** $\boldsymbol{M_y}$ (or M_z) in a cross-section is equal to the algebraic sum of the moments about the principal axis of the cross-section $\boldsymbol{Y}$ (or $\boldsymbol{Z}$) of all external forces applied to one of the cut-off parts.

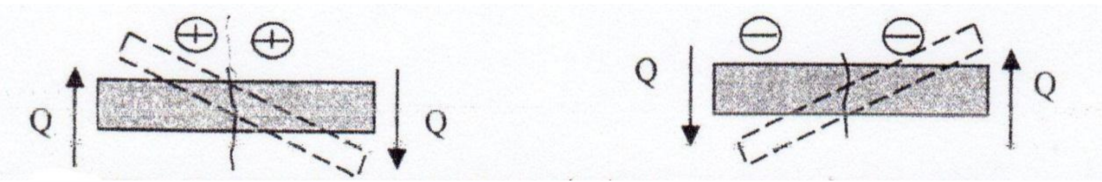
The **torque** $\boldsymbol{M_{tor}}$ in a cross-section is equal to the algebraic sum of the moments about the longitudinal axis $\boldsymbol{X}$ of the bar of all external forces applied to one of the cut-off parts.

If the bar's axis and the external loads lie in the same plane, only three of the six internal forces can be non-zero: the normal force $\boldsymbol{N}$, the shear force $\boldsymbol{Q_z} = \boldsymbol{Q}$, and the bending moment $\boldsymbol{My} = \boldsymbol{M}$. In this case, it is advisable to use the sign convention.

Sign Convention for Internal Forces in a Bar.

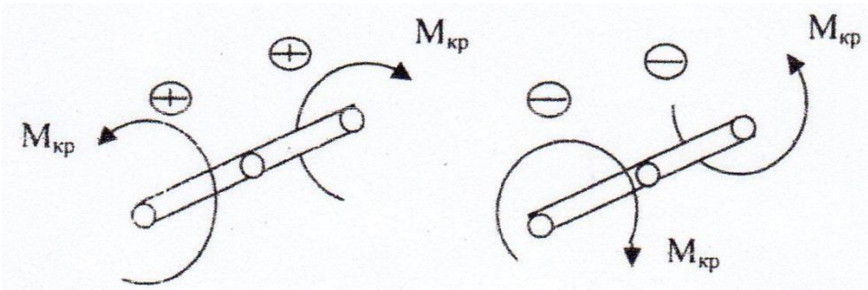
1. The normal force in a cross-section is positive if it causes tension in the cut-off part of the bar, and negative if it causes compression.

2. The shear force in a cross-section is positive if it tends to rotate the cut-off part of the bar clockwise relative to the section point, and negative if counter-clockwise.



3. The bending moment in a cross-section is positive if it causes tension in the lower fibers of the cut-off part of the bar, and negative if it causes tension in the upper fibers. The bending moment diagram is always plotted on the tension side.

4. The torque in a cross-section is positive if it rotates the cut-off part of the bar about the longitudinal axis X clockwise when viewed from the cross-section (i.e., from the end of the cut-off part), and negative if it rotates counter-clockwise



1.2 Task Execution Plan:

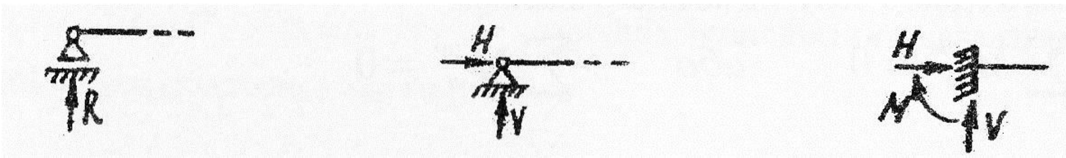
1. Drawing the calculation scheme;
2. Determination of support reactions;
3. Dividing the scheme into segments and determining characteristic cross-sections;
4. Calculation of internal forces at characteristic sections;
5. Construction of internal force diagrams;
6. Verification of the constructed diagrams.

2.3. Recommendations for Task Execution

1. In the schemes presented in Appendix 2, the bar is replaced by its axis (straight, broken, or curved line), on which lengths and loads are marked. The calculation scheme must be drawn strictly to scale, with all numerical data applied to it; the values of these data are indicated in Appendix 2 according to the student's cipher (variant).

Loads are applied to the longitudinal axis of the bar and are denoted by numerical values from Table D-2.

2. In calculation schemes of plane bar systems, three types of supports are encountered: Hinged-movable support (roller support) – has one constraint, in which one support reaction arises, acting along the supporting link (Fig. 2.2.a); Hinged-immovable support (pinned support) – has two linear constraints, in which two components of the support reaction arise (Fig. 2.2.b); Fixed support (clamped support) – imposes three constraints on the bar (two linear and one rotational), in which three reactions arise (two concentrated forces and a moment) (Fig. 2.2.c):



a b c

Fig. 2.2

All support reactions are assumed to be applied at the centroid of the support cross-section. The values of support reactions in statically determinate systems are determined from the equilibrium conditions of the entire system. Equilibrium equations for plane bar systems can be written in three forms:

— In the form of sums of projections onto two axes perpendicular to each other and the sum of moments of all forces about an arbitrary point lying in the plane of the scheme: $\sum x=0$; $\sum z=0$; $\sum M_o=0$.

— In the form of the sum of projections onto an axis and two sums of moments about points in the plane that do not lie on the same perpendicular to this axis of projections: $\sum x=0$; $\sum M_a=0$; $\sum M_в=0$.

— In the form of three sums of moments about three arbitrary points in the plane that do not lie on the same straight line: $\sum M_a=0$; $\sum M_в=0$; $\sum M_c=0$.

One or another variant of writing equilibrium equations (with the appropriate choice of points and axis directions) is used in each specific case in such a way as to avoid, if possible, solving a system of equations. To verify the correctness of determining support reactions, it is recommended to substitute their values into any equilibrium equation that was not used previously.

The direction of support reactions when writing equilibrium equations can be chosen arbitrarily. If during the calculation any reaction has a negative sign, one must change its direction in the drawing to the opposite and subsequently consider this reaction positive.

If a distributed load acts on the bar, then when determining reactions, it is replaced by a concentrated force equal to the area of the distributed load diagram and applied at the centroid of this diagram.

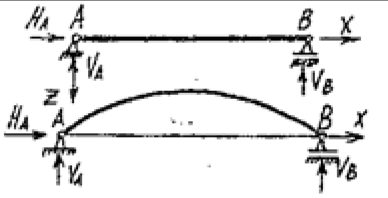
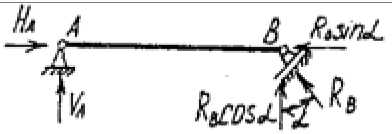
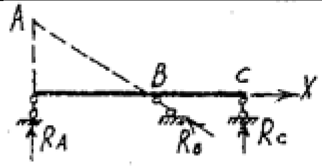
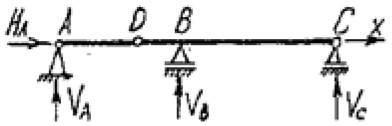
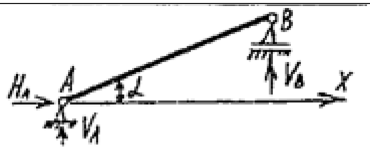
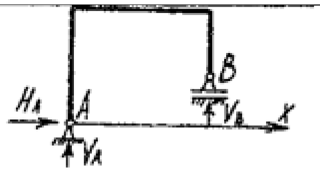
For hinged-cantilever beams, in addition to the ordinary equilibrium equations, it is necessary to additionally write an equation setting the sum of moments of all forces located to the left or right of each hinge to zero:

$$\sum M_h^{left} = 0 \qquad \text{або} \qquad \sum M_h^{right} = 0$$

Support reactions for beams with hinges can be determined by dividing them into simple beams.

A rational sequence for determining support reactions for some systems is presented in Table 2.1.

Table 2.1

№	Structural diagram	Equations for determining reactions	Determined reactions	Verification
1		$\sum M_A = 0$ $\sum M_B = 0$ $\sum X = 0$	V_B V_A H_A	$\sum Z = 0$
2		$\sum M_A = 0$ $\sum M_B = 0$ $\sum X = 0$	R_B V_A H_A	$\sum Z = 0$
3		$\sum M_A = 0$ $\sum M_B = 0$ $\sum X = 0$	R_C R_A R_B	$\sum Z = 0$
4		$\sum M_D = 0$ $\sum M_B = 0$ $\sum M_C = 0$	V_A V_C V_B	$\sum Z = 0$
5		$\sum M_A = 0$ $\sum Z = 0$ $\sum X = 0$	V_A V_B H_A	$\sum M_B = 0$
6		$\sum M_A = 0$ $\sum Z = 0$ $\sum X = 0$	V_A V_B H_A	$\sum M_B = 0$

In calculations of straight, curved, and broken bars that are rigidly fixed at one end, the determination of support reactions is not mandatory.

3. A segment (characteristic interval) of the system is defined as each part of it, within which the variations of internal forces remain constant. The boundaries of segments are cross-sections where concentrated forces (including support reactions) are applied, or where a distributed load begins and ends (or the load intensity changes according to a new law). In frames, the boundaries of segments are also nodal cross-sections.

Characteristic sections are those in which the values of forces are determined. Characteristic sections are the boundary points of all segments, and on segments with a uniformly distributed load — additionally 1–2 cross-sections.

4. Forces in characteristic sections can be calculated using written equations or without writing them. When writing equations, an arbitrary section is made in each segment of the bar, the origin of the coordinate ***X*** and the limits of its variation are indicated, and expressions (equations) of forces are written as functions of the coordinate ***X*** in accordance with the provided recommendations and sign conventions. The written expressions can be verified using differential relationships:

$$\frac{dN}{dx} = -q_x; \ \frac{dQ_y}{dx} = -q_y; \ \frac{dM_{кр}}{dx} = -m; \ \frac{dM_y}{dx} = \ Q_z; \ \frac{dM_z}{dx} = Q_y.$$

where $\mathbf{q_x}$; $\mathbf{q_y}$; $\mathbf{q_z}$; **m** – are the intensities of external distributed loads, the positive directions of which are indicated in Fig. 2.3.

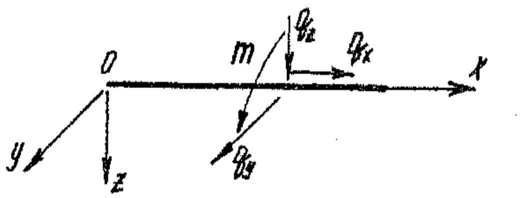


Fig. 2.3

It is more rational to calculate forces in characteristic sections directly, without writing equations.

In plane curved bars outlined by a circular arc of radius *R*, it is advisable to determine the position of an arbitrary section using polar coordinates. Then the internal forces **N, Q, M** will be functions of the angle φ.
The differential relationships between the forces have the form:

$$\frac{dN}{d\varphi} = Q - q_t \cdot R; \ \frac{dQ}{d\varphi} = -N - q_r \cdot R; \ \frac{dM}{d\varphi} = Q \cdot R.$$

where q_r and q_t – are the projections of the distributed load onto the radius and the tangent to the axis of the curved bar at the point of the section.

5. The values of N, Q_y, Q_z, M_{kr}, M_y, M_z, calculated in characteristic sections are the ordinates of the diagrams of the corresponding forces.

The diagram of any force is a graph that reflects the law of variation of the magnitude of this force along the length of the bar. Force diagrams for individual straight bars are constructed, each to its own scale, on axial lines that are parallel to the bar's axis, and for curved bars and frames – directly on the axes of the bar calculation scheme.

Ordinates are plotted perpendicular to the diagram axis: for M_y and M_z – on the side of the tension fiber, for other forces the choice of the ordinate direction is not critical. For individual horizontal bars loaded by a plane system of forces, it is advisable to use the following rule: positive ordinates of N, Q are plotted upwards, M – downwards, negative ones — conversely.

6. To verify force diagrams, primarily the rules derived from the differential relationships presented in Item 4 are used. In particular, for a plane system loaded in its plane, based on the relationships:

$$\frac{dQ_z}{dx} = -q_z; \quad \frac{dM_y}{dx} = Q_z.$$

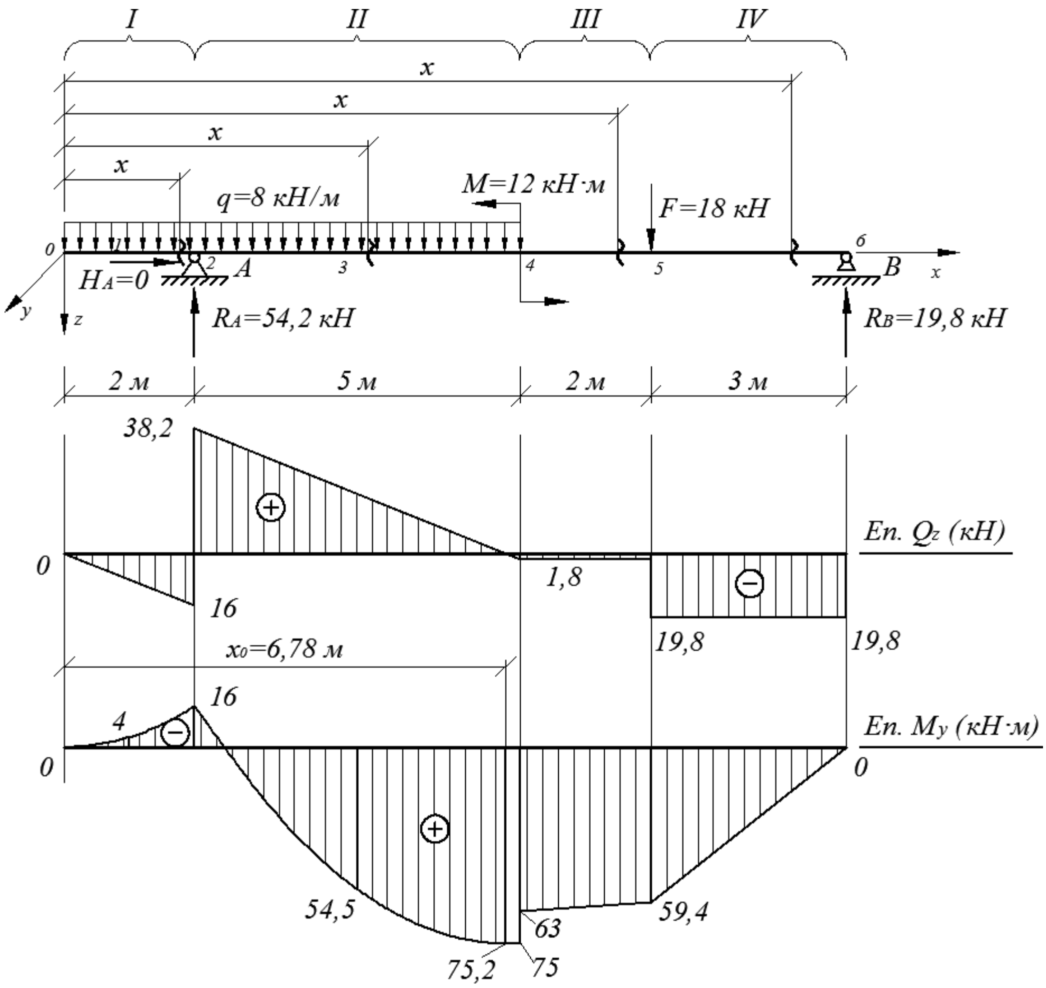
one can determine: based on the character of the load – the outlines of the Q_z and M_y diagram lines; based on the value of the shear force – the steepness of the slope of the line bounding the bending moment diagram; based on the sign of the shear force Q_z – the increase or decrease of the ordinates of the M_y diagram; based on the zero points of the Q_z diagram – the presence of extrema in the M_y diagram.

Points of application of concentrated forces correspond to kinks in the M_y diagram in the direction of the force action, and jumps in the Q_z diagram by the magnitude of these forces; points of application of concentrated moments correspond to jumps in the M_y diagram by the magnitude of the moments; the character of the Q_z diagram in this case does not change.

To verify the construction of force diagrams, one can also use the equilibrium conditions of the cut-off part of the system, by applying external loads to it, and in the made sections – the forces taken from the corresponding diagrams. In frames, it is advisable to check the equilibrium of cut-out nodes.

Examples

<u>Example 1</u>. A horizontal beam on two supports subjected to an external load applied in the plane of the beam (Fig. 2.4).



Determination of support reactions:

$\sum X = 0;\ H_A = 0$;.

$\sum M_A = 0;$

$R_B * 10 - 8 * 7 * 1{,}5 + 12 - 18 * 7 = 0;\ R_B = 19{,}8$ кН;

$\sum M_B = 0;$

$-R_A * 10 + 8 * 7 * 8{,}5 + 12 + 18 * 3 = 0;\ R_A = 54{,}2$ кН;

Verification: $\sum Z = 0;\ 8 * 7 - 54{,}2 + 18 - 19{,}8 = 0.$

We take the origin of the coordinate X at the left end of the beam. We divide the beam into segments I, II, III, IV and determine the characteristic cross-sections:

$$0, 1, 2, \dots ,6\ (x = 0; 1; 2; 4; 5; 7; 9; 12 \text{ м}).$$

Segment I $(0 \le x \le 2 \text{ м})$

We write expressions for Q and M,

$$Q = -8 \cdot x;\ M = -8 \cdot x\frac{x}{2} = -4 \cdot x^2.$$

Segment II $(2 \text{ м} \le x \le 7 \text{ м})$

$$Q = -8 \cdot x + 54{,}2;$$

$$M = -8 \cdot x\frac{x}{2} + 54{,}2 \cdot (x - 2) = -4 \cdot x^2 + 54{,}2x - 108{,}4.$$

Segment III $(7 \text{ м} \le x \le 9 \text{ м})$

$$Q = -8 \cdot 7 + 54{,}2 = 1{,}8 \text{ кН}.$$

$$M = -8 \cdot 7\left(x - \frac{7}{2}\right) + 54{,}2 \cdot (x - 2) = -56 \cdot (x - 3{,}5) + 54{,}2(x - 2) - 12.$$

Segment IV $(9 \text{ м} \le x \le 12 \text{ м})$

$$Q = -8 \cdot 7 + 54{,}2 - 18 = -19{,}8 \text{ кН}.$$

$$M = -8 \cdot 7(x - 3{,}5) + 54{,}2 \cdot (x - 2) - 12 - 18 \cdot (x - 9) = 0.$$

We determine the values of shear forces and bending moments at characteristic points on the beam by substituting into the obtained equations the values of the coordinate X of the beginning and end of characteristic segments; we summarize the results in a table.

X[m]	0	1	2	2	4,5	max	7	7	9	9	12
Q[кН]	0	-	-16	38,2	-	0	-1,8	-1,8	-1,8		-19,8
M[кНm]	0	-4	-16	-16	-54,5	75,2	75	63	59,4	59,4	0

Based on the calculation results, we construct the Q and M diagrams.

In segment II, it is necessary to determine the extremal point of the bending moment. When Q = 0, M = max.

$Q = -8 \cdot x_0 + 54{,}2 = 0 \rightarrow x_0 = 6{,}78$ m.

$M_{max} = -4 \cdot 6{,}78^2 + 54{,}2 \cdot (6{,}78 - 2) = 75{,}2$ кН · m.

<u>Example 2</u>. A beam on two supports with a cantilever, loaded by an inclined uniformly distributed load. (Fig. 2.5)

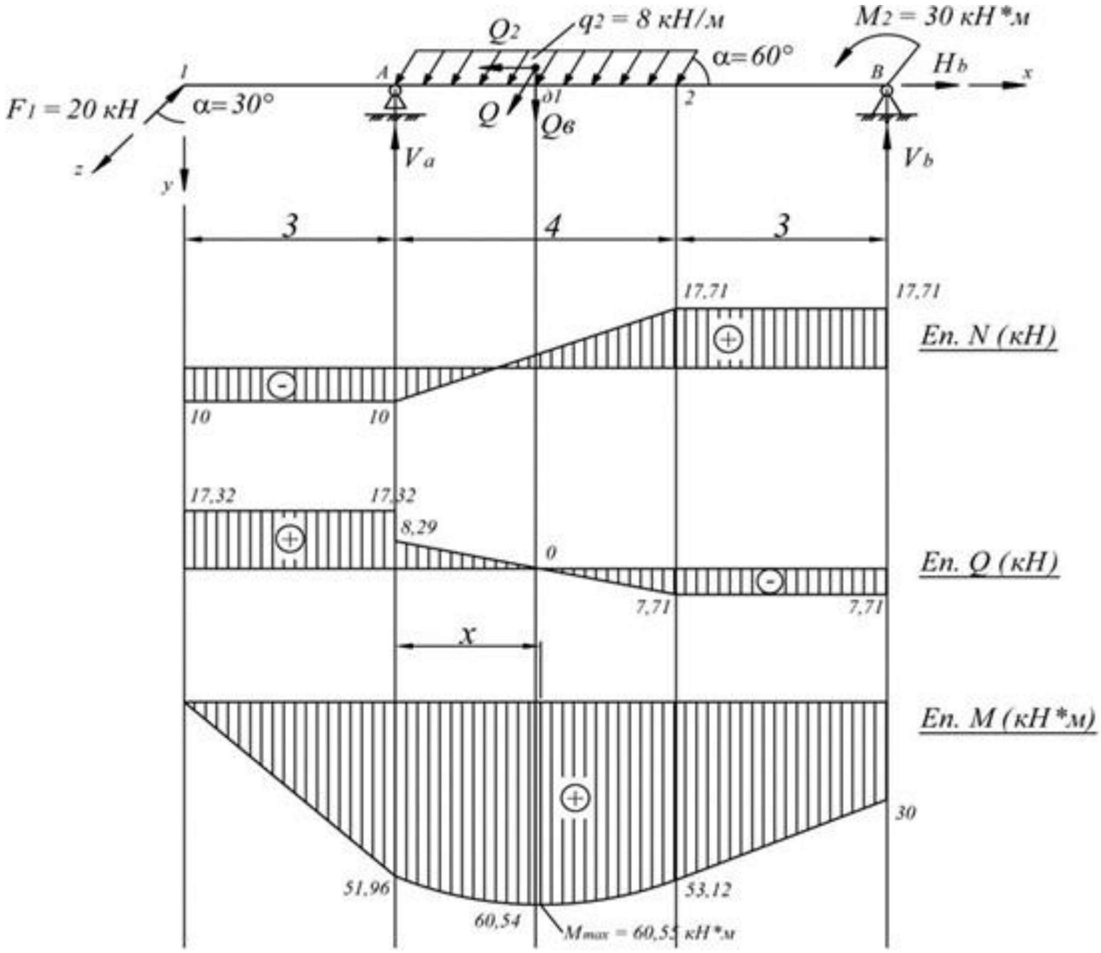


Fig. 2.5

We replace the uniformly distributed load with a resultant force and resolve the inclined forces into two components – vertical and horizontal:

$$Q = q_2 \cdot 4 = 8 \cdot 4 = 32 \text{ кН}$$

$$Q^2 = Q \cdot \sin 60^{\circ} = 32 \cdot \frac{\sqrt{3}}{2} = 27{,}11 \text{ кН}$$

$$Q^B = Q \cdot \cos 60^{\circ} = 32 \cdot \frac{1}{2} = 16 \text{ кН}$$

$$F_1^B = F_1 \cdot \cos 30^{\circ} = 20 \cdot \frac{\sqrt{3}}{2} = 17{,}32 \text{ кН}$$

$$F_1^2 = F_1 \cdot \sin 30^{\circ} = 20 \cdot \frac{1}{2} = 10 \text{ кН}$$

We determine the support reactions

$\sum X = 0;$

$$F_2 - Q_2 + H_b = 0;$$

$$10 - 27{,}71 = -H_b; H_b = 17{,}71 \text{ кН};$$

$\sum \text{M}_\text{a} = 0;$

$$F^{\text{в}} * 3 + Q^{\text{в}} * 2 - M_2 - V_b * 7 = 0$$

$$V_b = \frac{-17{,}32 \cdot 3 + 16 \cdot 2 - 30}{7} = 7{,}71 \text{ кН}.$$

$\sum \text{M}_b = 0;$

$$F^{\text{в}} * 10 + V_a * 7 - Q^{\text{в}} * 5 - M_2 = 0$$

$$V_\text{a} = \frac{-17{,}32 \cdot 10 + 16 \cdot 5 + 30}{7} = -9.03 \text{ кН}.$$

Verification: $\sum Y = 0; F^{\text{в}} + V_a - Q^{\text{в}} + V_b = 17{,}32 + (-9{,}03) - 16 + 7{,}71 = 0$

We take the origin of the coordinate x, mark the characteristic points on the beam, and determine the ordinates of the M, Q, N force diagrams (Fig. 2.5) at characteristic and additional points using the method of sections, considering the part of the beam to the left of the section.

Axial force: (x)

$N_1^{л} = 0.$

$N_1 = -F_1^2 = -10$ кН.

$N_a = -F_1^2 = -10$ кН.

$N_2 = -F_1^2 + Q^2 = -10 + 27{,}71 = 17{,}71$ кН.

$N_в^{л} = -F_1^2 + Q^2 = -10 + 27{,}71 = 17{,}71$ кН.

$N_п^{л} = -F_1^2 + Q^2 - H_b = -10 + 27{,}71 - 17{,}71 = 0.$

Shear force: (z)

$Q_1^{л} = 0.$

$Q_1^{п} = F_1^2 = 17{,}32$ кН.

$Q_а^{л} = F_1^2 = 17{,}32$ кН.

$Q_а^{п} = F_1^2 + V_a = 17{,}32 + (-9{,}03) = 8{,}29$ кН.

$Q_2 = F_1^2 + V_a - Q^{в} = 17{,}32 + (-9{,}03) - 16 = -7{,}71$ кН.

$Q_в^{л} = F_1^2 + V_a - Q^{в} = 17{,}32 + (-9{,}03) - 16 = -7{,}71$ кН.

$Q_в^{п} = F_1^2 + V_a - Q^{в} + V_b = 17{,}32 + (-9{,}03) - 16 + 7{,}71 = 0.$

Bending moment: M_y

$M_1 = 0$

$M_a = F_1^{в} \cdot 3 = +17{,}32 \cdot 3 = 51{,}96$ кН · m.

$M_1 = F_1^{в} \cdot 5 + V_a \cdot 2 - q_2 \cdot \cos 60^{\circ} \cdot 2 \cdot 1 = 17{,}32 \cdot 5 + (-9{,}03) \cdot 2 -$

$-8\frac{1}{2} \cdot 2 \cdot 1 = 60{,}54$ кН · m;

$M_2 = F_1^{в} \cdot 7 + V_a \cdot 4 - q_2 \cdot \cos 60^{\circ} \cdot 4 \cdot 2 = 17{,}32 \cdot 7 + (-9{,}03) \cdot 4 -$

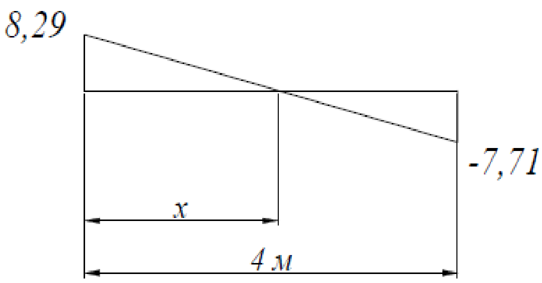
$-8\frac{1}{2} \cdot 4 \cdot 2 = 53{,}12$ кН · m;

$M_b^{л} = F_1^{в} \cdot 10 + V_a \cdot 7 - q_2 \cdot \cos 60^{\circ} \cdot 4 \cdot 5 = 17{,}32 \cdot 10 + (-9{,}03) \cdot 7 -$

$-8\frac{1}{2} \cdot 4 \cdot 5 = 30$ кН · m;

$M_b^{п} = M_b^{л} - M_2 = 30 - 30 = 0$ кН · m;

Determination of the position of M_{max} on the Moment Diagram. Let us isolate the segment where $Q = 0$.



$8{,}29 + 7{,}71 \;\rightarrow 4$

$8{,}29 \;\rightarrow x$

$$x = \frac{8{,}29 \cdot 4}{16} = 2{,}07 \text{ m}$$

We determine , by considering a section at a distance x = 2,07 m from support A.

$$M_{max} = F_1^{в} \cdot 5{,}07 + V_a \cdot 2{,}07 - q_2 \cdot \cos 60^{\circ} \cdot 2{,}07 \cdot \frac{2{,}07}{2} =$$

$$= 17{,}32 \cdot 5{,}07 + (-9{,}03) \cdot 2{,}07 - 8\frac{1}{2} \cdot 2{,}07 \cdot \frac{2{,}07}{2} = 60{,}55 \text{ кН}.$$

Example 3. Cantilever loaded by inclined forces.

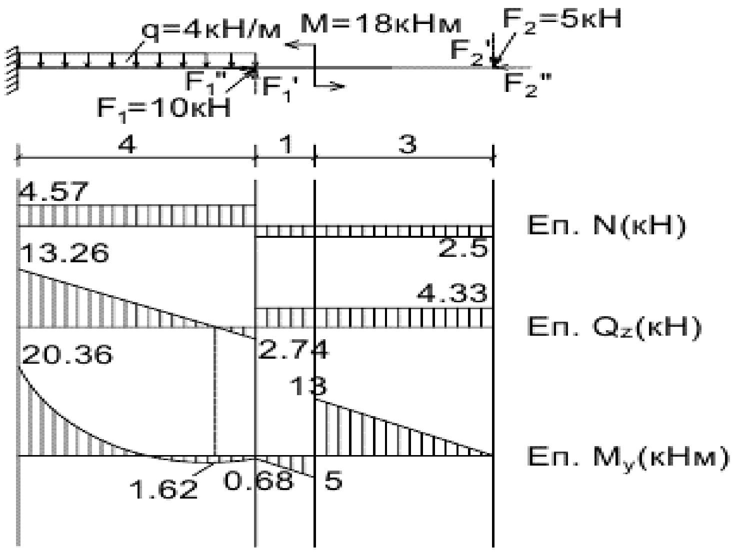


Fig. 2.6

We resolve each inclined force into two components: vertical and horizontal:

$$F_1^{\text{в}} = F_1 \cos 45^{\circ} = 10 \cdot 0{,}707 = 7{,}07 \text{ кН};$$

$$F_1^{\text{г}} = F_1 \sin 45^{\circ} = 10 \cdot 0{,}707 = 7{,}07 \text{ кН}$$

$$F_1^{\text{в}'} = F_1 \sin 45^{\circ} = 10 \cdot 0{,}707 = 7{,}07 \text{ кН}$$

$$F_1^{\text{г}'} = F_1 \sin 45^{\circ} = 10 \cdot 0{,}707 = 7{,}07 \text{ кН}$$

$$F_2^{\text{в}} = F_2 \cos 30^{\circ} = 5 \cdot 0{,}866 = 4{,}33 \text{ кН};$$

$$F_2^{\text{г}} = F_2 \cos 30^{\circ} = 5 \cdot 0{,}866 = 4{,}33 \text{ кН};$$

$$F_2^{\text{в}''} = F_2 \sin 30^{\circ} = 5 \cdot 0{,}5 = 2{,}5 \text{ кН}$$

$$F_2^{\text{г}''} = F_2 \sin 30^{\circ} = 5 \cdot 0{,}5 = 2{,}5 \text{ кН}$$

In this case, we do not calculate the support reactions at the fixed support. We determine the forces by considering the conditionally cut-off right part of the beam, without writing equations, at characteristic sections.

Axial force $N_{(x)}$:

$N_0 = -2{,}5$ кН;

$N_{2П} = -2{,}5$ кН;

$N_{2Л} = -2{,}5 + 7{,}07 = 4{,}57$ кН;

$N_4 = 4{,}57$ кН.

Shear force $Q_{(x)}$:

$Q_0 = 4{,}33$ кН;

$Q_{2П} = 4{,}33$ кН;

$Q_{2Л} = 4{,}33 - 7{,}07 = -2{,}74$ кН;

$Q_4 = 4{,}33 - 7{,}07 + 4 \cdot 4 = 13{,}26$ кН.

Bending moment $M_{y(x)}$:

$M_0 = 0$;

$M_{1П} = -4{,}33 \cdot 3 = -13$ кН $\cdot$ m;

$M_{1Л} = -4{,}33 \cdot 3 + 18 = 5$ кН $\cdot$ m;

$M_2 = -4{,}33 \cdot 4 + 18 = -0{,}68$ кН $\cdot$ m;

$M_3 = -4{,}33 \cdot 6 + 18 + 7{,}07 \cdot 2 - 4 \cdot 2 \cdot 1 = -1{,}84$ кН $\cdot$ m;

$$M_4 = -4{,}33 \cdot 8 + 18 + 7{,}07 \cdot 4 - 4 \cdot 2 \cdot 2 = -20{,}36 \text{ кН} \cdot \text{m};$$

$$M_{max} = -4{,}33 \cdot 4{,}69 + 18 + 7{,}07 \cdot 0{,}69 - 4\frac{0{,}69^2}{2} = 1{,}62 \text{ кН} \cdot \text{m};$$

We construct the internal force diagrams: N, Q_z, M_y Fig. 2.6

<u>Example 4</u>. The beam is attached to the base at three points by means of supporting rods (Fig. 2.7).

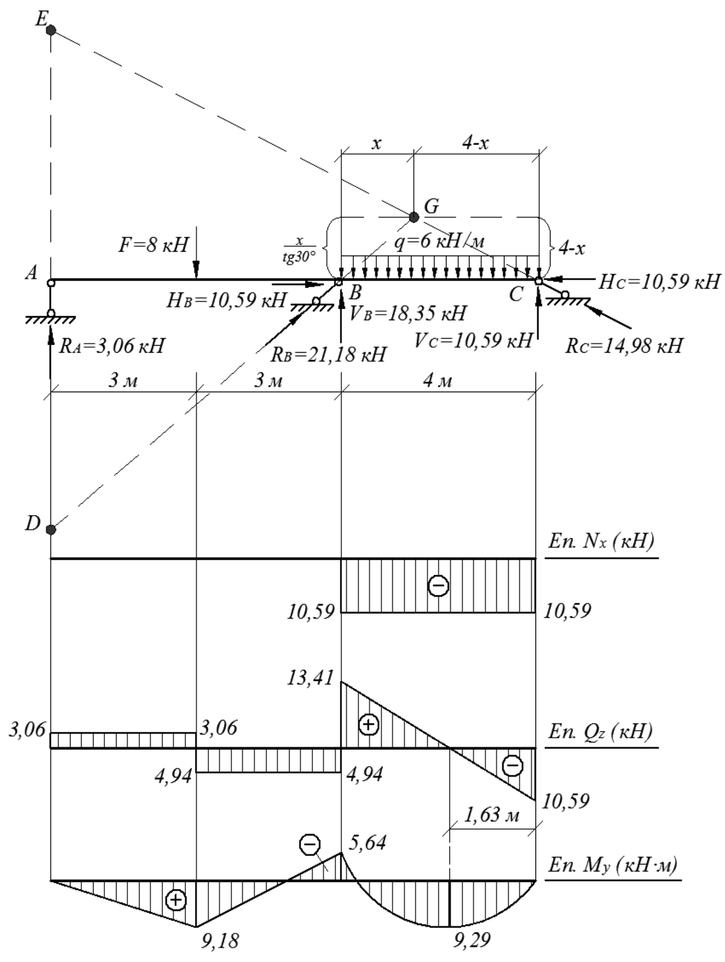


Fig. 2.7

The support reactions act along the rods, whose lines of action do not intersect at a single point. Therefore, this system is geometrically invariant.

The intersection points D and E of the support reactions R_a with R_b and R_a with R_c lie on a vertical line passing through point A and are located at distances from it, respectively:

$$AD = 6 \cdot \text{ctg}\,30^{\circ} = 6 \cdot 1{,}733 = 10{,}39 \text{ м; i } AE = 10 \cdot \text{ctg}\,45^{\circ} = 10 \cdot 1 = 10 \text{ m}$$

We determine the position of the intersection point G the support reactions R_B and R_C .

$$\frac{x}{\text{ctg}\,30^{\circ}} = 4 - x; x = (4 - x) \cdot 0{,}577; x = 1{,}46 \text{ m}$$

$$\sum M_D = 0;\ R_c \cdot \cos 45^{\circ} \cdot 10 + R_c \sin 45^{\circ} \cdot 10{,}39 - 8 \cdot 3 - 6 \cdot 4 \cdot 8 = 0;$$

R_c =14,98 кН; $V_c = R_c \cos 45^{\circ} = 14{,}98 \cdot 0{,}707 = 10{,}59$ кН.

$$H_c = R_c \sin 45^{\circ} = 14{,}98 \cdot 0{,}707 = 10{,}59 \text{ кН.}$$

$$\sum M_E = 0;\ R_b \cdot \cos 30^{\circ} \cdot 6 + R_b \sin 30^{\circ} \cdot 10 - 8 \cdot 3 - 6 \cdot 4 \cdot 8 = 0;$$

R_b =21,18 кН; $V_b = R_b \cdot \sin 30^{\circ} = 21{,}18 \cdot 0{,}5 = 10{,}59$ кН.

$$\sum M_G = 0;\ R_A \cdot 7{,}46 + 8 \cdot 4{,}46 - 6 \cdot 4 \cdot 0{,}54 = 0; R_A = 3{,}06 \text{ кН.}$$

Verification:

$\sum F_x = 0;\quad 10{,}59 - 10{,}59 = 0;$

$\sum F_z = 0;\quad 3{,}06 + 18{,}35 + 10{,}59 - 8 - 6 \cdot 4 = 0.$

We calculate and construct the internal force diagrams: N, Q_z, M_y Fig. 2.6

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Example 5. Hinged-cantilever beam (Fig.2.8).

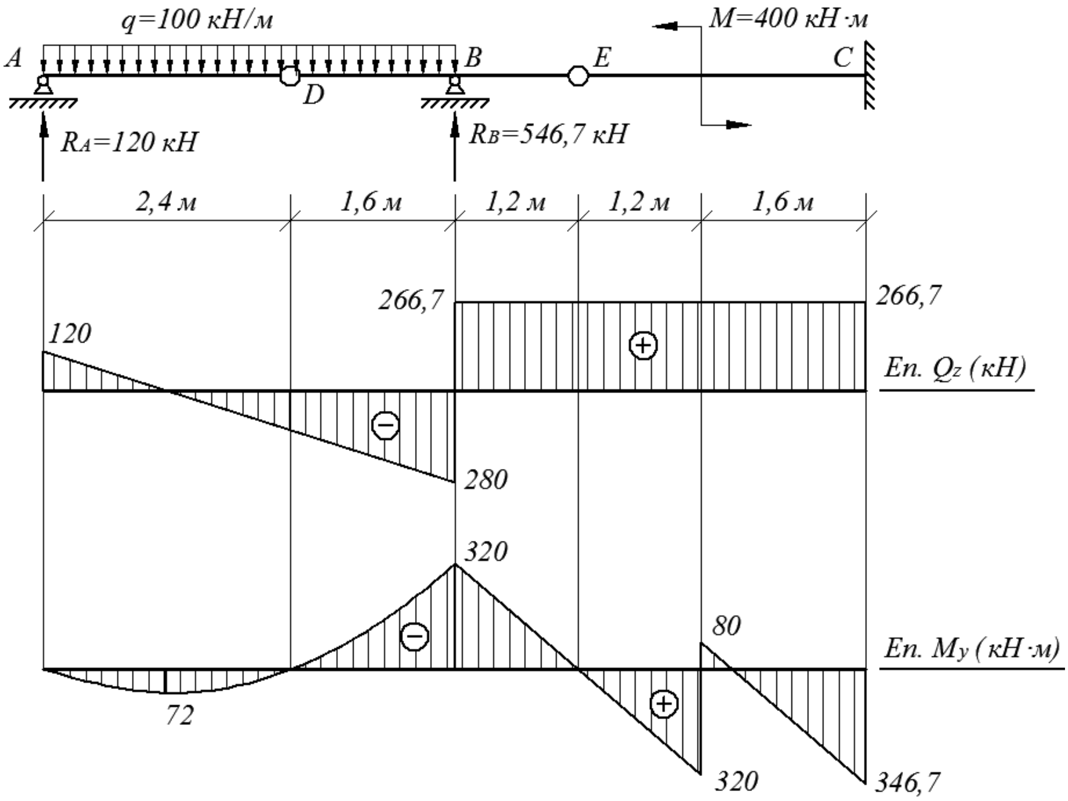


Fig.2.8

$$\sum M_D^l = 0;\ -R_A \cdot 2{,}4 + 100 \cdot 2.4 \cdot 1{,}2 = 0;\ R_A = 120\ kH.$$

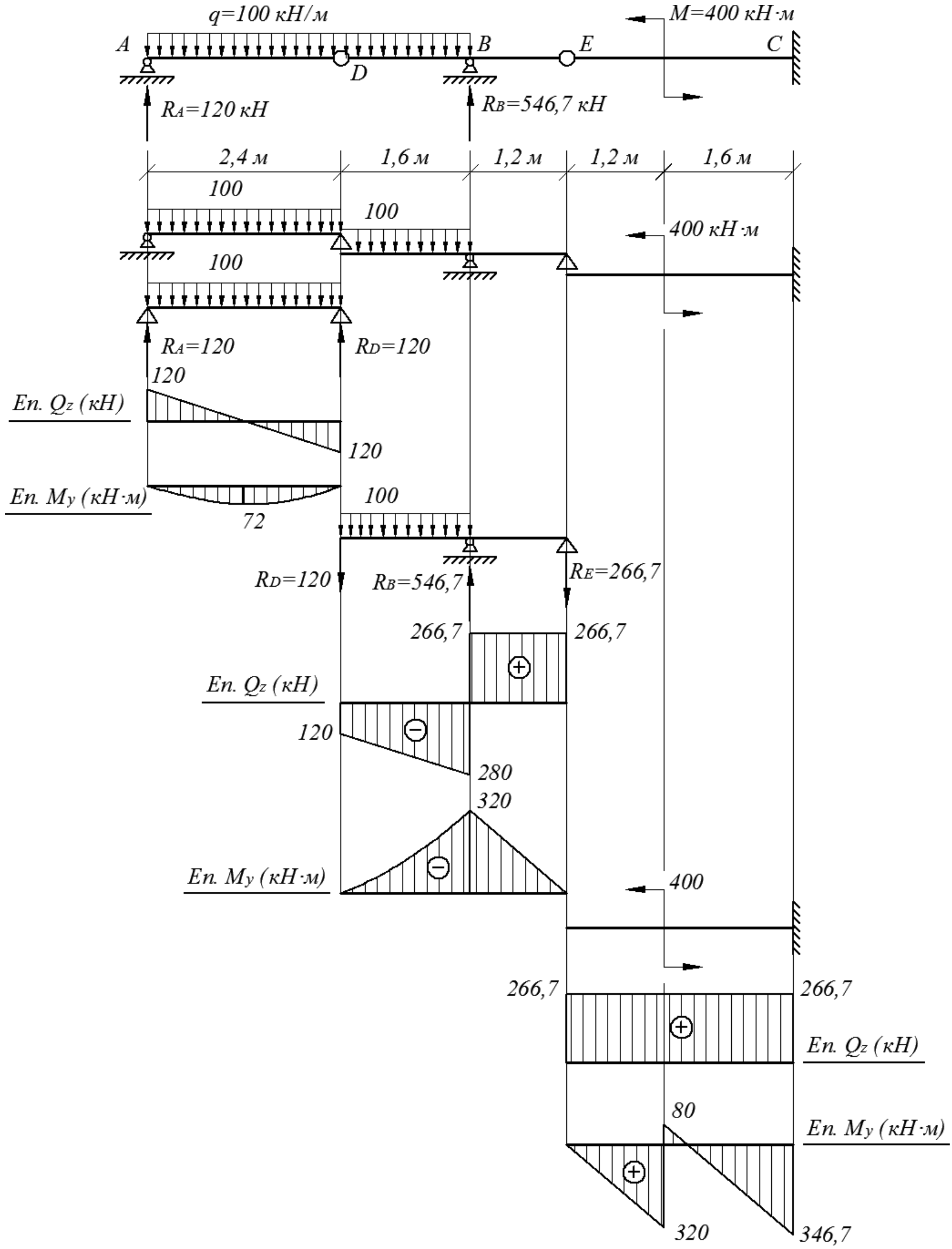
$$\sum M_E^l = 0;\ -R_B \cdot 1{,}2 + 100 \cdot 4 \cdot 3{,}2 - 120 \cdot 5{,}2 = 0;\ R_B = 546{,}7\ kH.$$

$$\sum M_c = 0;\ M_c + 400 - 120 \cdot 8 + 100 \cdot 4 \cdot 6 - 546{,}7 \cdot 4 = 0;\ M_c = 346{,}7\ kH\text{m}.$$

$$\sum F_y = 0;\ R_c - 120 - 546{,}7 + 100 = 0;\ R_c = 266{,}7\ kH\text{m}.$$

We determine the ordinates of the force diagrams, considering the external loads located to the left of the calculation sections, and construct the diagrams Q_z, M_y Fig. 2.8.

We calculate this system by dividing it into separate beams. The sequence for determining reactions and constructing force diagrams using this method is shown in Fig. 2.9.



q=100 кН/м
M=400 кН·м
A
B
E
C
D
RA=120 кН
RB=546,7 кН
2,4 м
1,6 м
1,2 м
1,2 м
1,6 м
100
100
400 кН·м
100
RA=120
RD=120
120
Еп. Qz (кН)
120
Еп. My (кН·м)
72
100
RD=120
RB=546,7
RE=266,7
266,7
266,7
Еп. Qz (кН)
120
280
320
Еп. My (кН·м)
400
266,7
266,7
Еп. Qz (кН)
80
Еп. My (кН·м)
320
346,7

Fig. 2.9

<u>Example 6</u>. A plane frame subjected to a load acting in the plane of the frame (Fig. 2.10). First, we determine the angle α and calculate its trigonometric functions.

$$\tan\alpha = \frac{3}{4} = 0{,}75;\ \alpha = 36{,}87^{\circ};\ \sin\alpha = 0{,}6;\ \cos\alpha = 0{,}8.$$

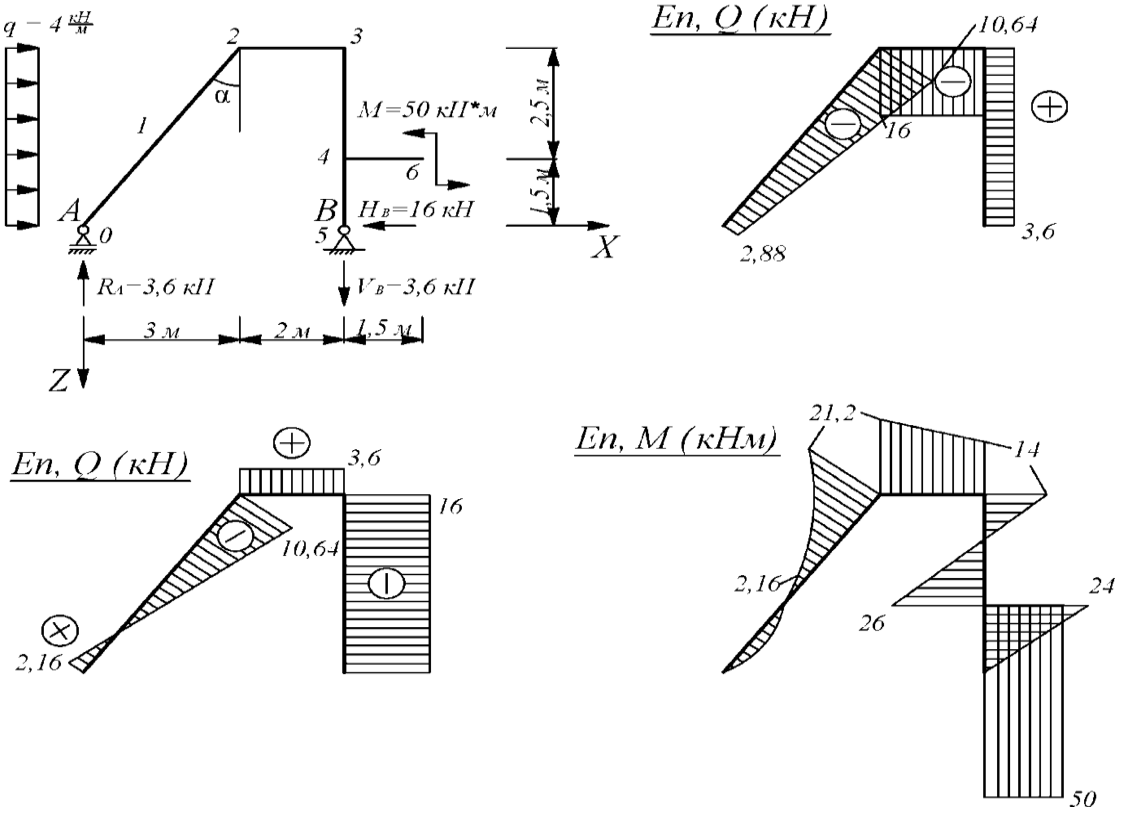


Fig. 2.10

We determine the support reactions:

$\sum M_b = 0;\ \ -R_a * 5 - 4 * 4 * 2 + 50 = 0;\ \ R_a = 3{,}6$ kH;

$\sum M_a = 0;\ \ -V_b * 5 - 4 * 4 * 2 + 50 = 0; V_b = 3{,}6$ kH;

$\sum X = 0;\ \ -H_b + 4 * 4 = 0;\ H_b = 16$ kH;

Verification:

$\sum Z = 0;\ -3{,}6 + 3{,}6 = 0.$

We determine the characteristic cross-sections and calculate the values of internal forces in them.

Axial force:

$N_0 = -R_a * \cos\alpha = -3,6 * 0,8 = -2,88\ kH;$

$N^H = -R_a * \cos\alpha - q * 4 * \sin\alpha = -3,6 * 0,8 - 4 * 4 * 0,6 = -12,48\ kH;$

$N_{2-3} = -q * 4 = -4 * 4 = -16\ kH;$

$N_{3-5} = V_a = 3,6\ kH;$

$N_{4-6} = 0$

Shear force:

$Q_0 = R_a * \sin\alpha = 3,6 * 0,6 = 2,16\ kH;$

$Q^H = R_a * \sin\alpha - q * 4 * \cos\alpha = 3,6 * 0,6 - 4 * 4 * 0,8 = -10,64\ kH;$

$Q_{п}^{2} = Q_{л}^{3} = R_a = 3,6\ kH;$

$Q_3^{\text{Н}} = Q_5^{\text{В}} = H_{\text{В}} = 16\ kH;$

$Q_4^{\Pi} = Q_6 = 0.$

Bending moment:

$M_0^B = 0.$

$$M_1 = R_a \cdot \frac{3}{2} - q \cdot 2 \cdot \frac{2}{2} = 3,6 \cdot 1,5 - 4 \cdot 2 \cdot 1 = 2,6\ kH \cdot m;$$

$$M_2^H = R_a \cdot 3 - q \cdot 4 \cdot \frac{4}{2} = 3,6 \cdot 3 - 4 \cdot 4 \cdot 2 = -21,2\ kH \cdot m;$$

$$M_3^{Л} = M_5^{\Pi} = R_a \cdot 5 - q \cdot 4 \cdot \frac{4}{2} = 3,6 \cdot 5 - 4 \cdot 4 \cdot 2 = -14\ kH \cdot m;$$

$M_4^{\Pi} = M_6^{Л} = -50\ kH \cdot m;$

$M_4^B = H_B \cdot 1,5 - 50 = 16 \cdot 1,5 - 50 = -26\ kH \cdot m;$

$M_4^H = H_B \cdot 1,5 = 16 \cdot 1,5 = 24\ kH \cdot m;$

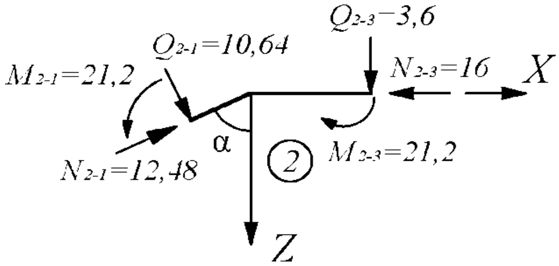
$M_5^B = 0.$

We calculate and construct the internal force diagrams: N, Q_z, M_y Fig. 2.10

We perform an equilibrium check of the isolated nodes 2, 3, and 4 by applying the acting internal forces to them.

<u>Node 2</u>

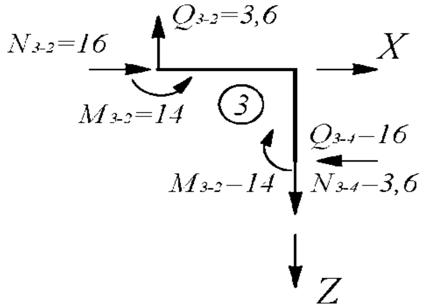
$$\sum X = 0;\ \ 10{,}64 \cdot 0{,}8 + 12{,}48 \cdot 0{,}6 - 16 = 0;$$



$$\sum Z = 0;\ \ 10{,}64 \cdot 0{,}6 - 12{,}48 \cdot 0{,}8 + 3{,}6 = 0;$$

$$\sum M = 21{,}2 - 21{,}2 = 0.$$

<u>Node 3</u>

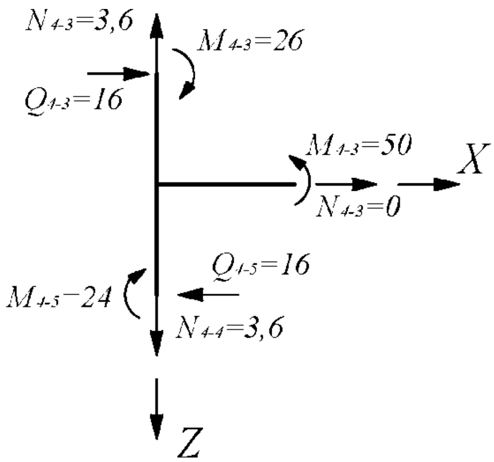


$$\sum X = 16 - 16 = 0;$$

$$\sum Z = 3{,}6 - 3{,}6 = 0;$$

$$\sum M = 14 - 14 = 0.$$

<u>Node 4</u>



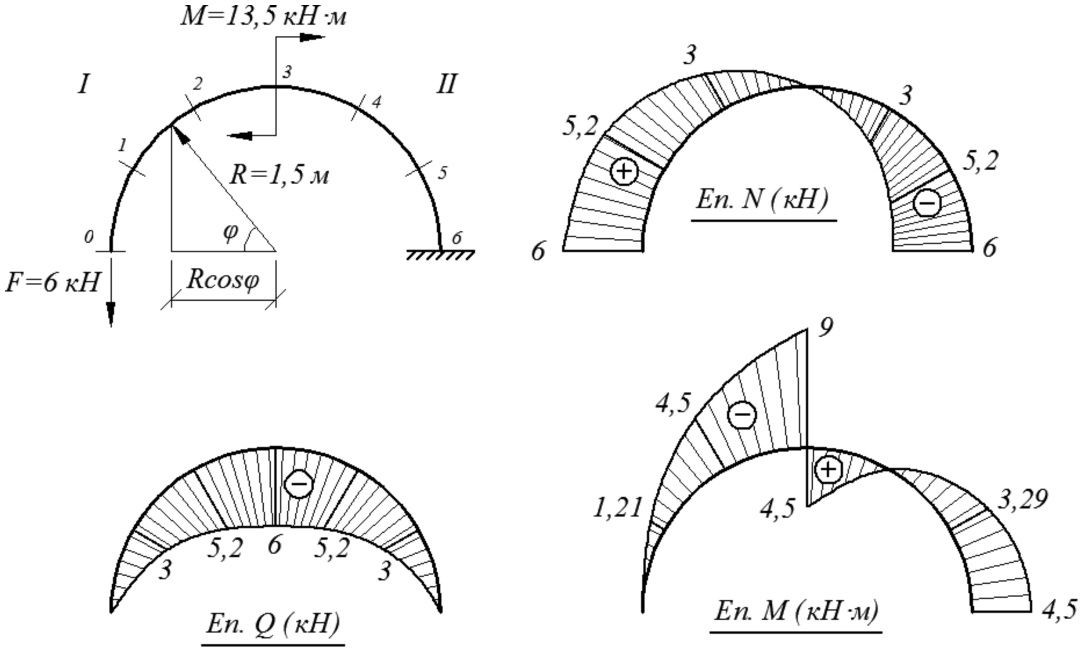
$$\sum X = 16 - 16 = 0;$$

$$\sum Z = 3{,}6 - 3{,}6 = 0;$$

$$\sum M = 50 - 26 - 24 = 0.$$

Example 7. A plane curved bar, the axis of which is a circular arc (Fig. 2.11).

Fig. 2.11



We do not determine the reactions at the fixed support. We divide the bar axis into two segments and write equations to determine the internal forces. Considering the left conditionally cut-off part of the bar, we project the forces onto the tangent and the normal to the bar axis and determine the sum of moments relative to the center of the cross-section.

Segment I $\left(0 \le \varphi \le \frac{\pi}{2}\right)$

$$N(\varphi) = 6 \cdot \cos\varphi;$$

$$Q(\varphi) = -6 \cdot \sin\varphi;$$

$$M(\varphi) = -6 \cdot 1{,}5 \cdot (1 - \cos\varphi) = -9 \cdot (1 - \cos\varphi).$$

Segment II $\left(\frac{\pi}{2} \le \varphi \le \pi\right)$

$$N(\varphi) = 6 \cdot \cos\varphi;$$

$$Q(\varphi) = -6 \cdot \sin\varphi;$$

$$M(\varphi) = -9 \cdot (1 - \cos\varphi) + 13{,}5.$$

The calculations of N, Q, M in sections with an interval of 30° are presented in the table

Segment	φ, *degrees*	sinφ	cosφ	$N(\varphi)$, *кН*	$Q(\varphi)$, *кН*	$M(\varphi)$, *кНм*
I	0	0	1	6	0	0
	30°	0,5	0,866	5,2	-3	-1,21
	60°	0,866	0,5	3	-5,2	-4,5
	90°	1	0	0	-6	-9
II	90°	1	0	0	-6	4,5
	120°	0,866	-0,5	-3	-5,2	0
	150°	0,5	-0,866	-5,2	-3	-3,29
	180°	0	-1	-6	0	-4,5

We construct the diagrams of N, Q, M, by plotting the ordinates along the normal to the curved axis (Fig. 2.11)

Example 8. A space frame fixed at one end, consisting of three mutually perpendicular bars (Fig. 2.12)

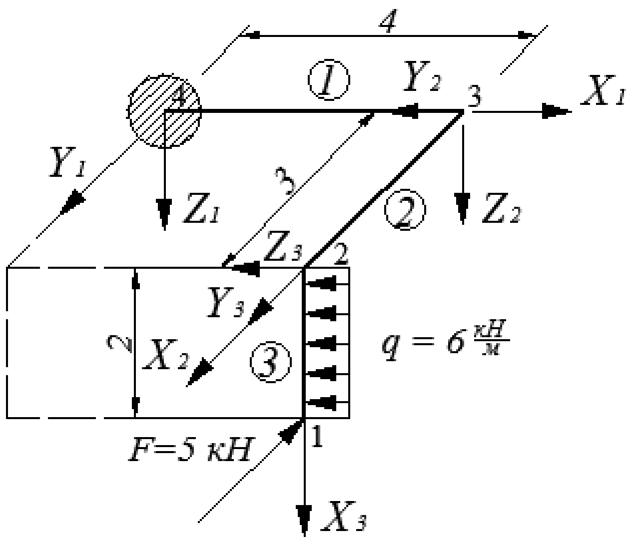
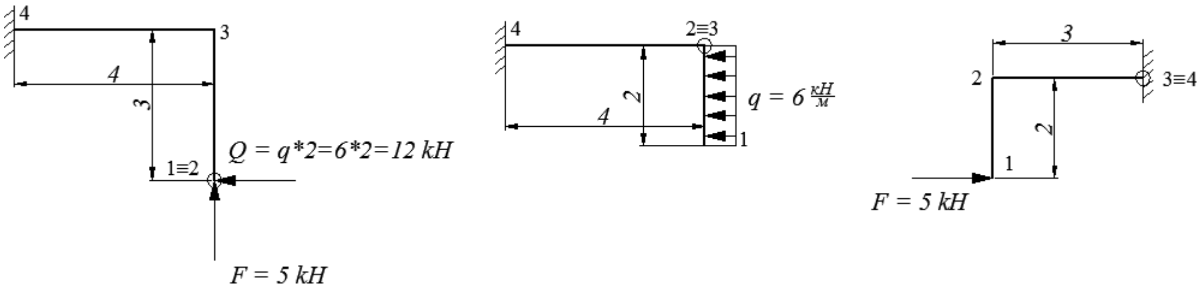


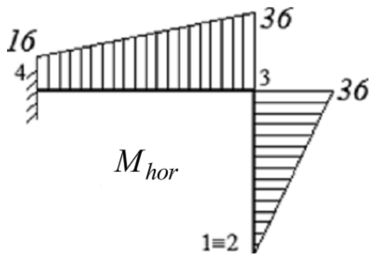
Fig. 2.12

The construction of internal force diagrams can be carried out using two methods:

Method 1: Project the space structure onto the horizontal, frontal, and profile coordinate planes together with the applied load. Construct the diagrams of bending moments M, shear forces Q, and axial forces N for each plane



Horizontal Frontal Profile

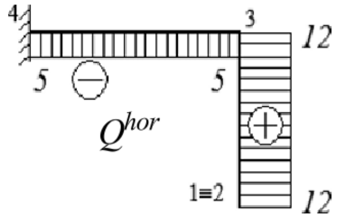


$$M_{1-2} = 0$$

$$M^{H} = Q * 3 = 12 * 3 = 36\ kH \text{ *m}$$

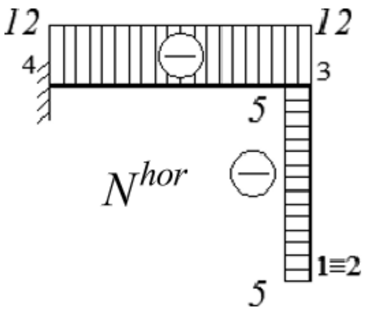
$$M^{Л} = Q * 3 = 12 * 3 = 36\ kH \text{ *m}$$

$$M_4 = Q * 3 - F * 4 = 12 * 3 - 5 * 4 = 16\ kH * m$$



$$Q_{1-2} = Q^{H} = Q = 12\ kH$$

$$Q^{Л} = Q_4 = -F = -5\ kH$$

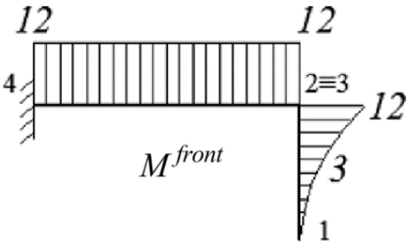


$$N_{1-2} = N^{H} = -F = -5\ kH$$

$$N^{Л} = N_4 = -Q = -12\ kH$$

Fig. 2.13 Diagrams of bending moments (a), shear forces (b), and axial forces (c) in the horizontal projection.

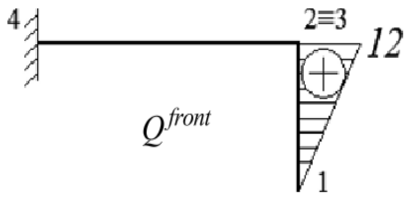
$$M_1 = 0$$



$$M_{2-3}^{H} = M_{2-3}^{Л} = q \cdot 2 \cdot \frac{2}{2} = 6 \cdot 2 \cdot 1 = 12\ kHm.$$

$$M_{q1} = q * 1 * 0{,}5 = 6 * 1 * 0{,}5 = 3\ kH * m$$

$$M_4 = q * 2 * 1 = 6 * 2 * 1 = 12\ kH * m$$

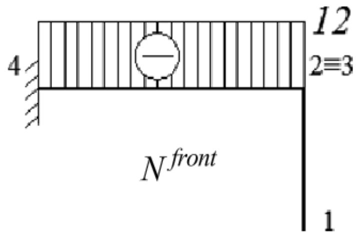


$$Q_1 = 0$$

$$Q^{H} = q * 2 = 6 * 2 = 12\ kH * m$$

$$Q_{2-3}^{Л} = 0$$

$$Q_4 = 0$$



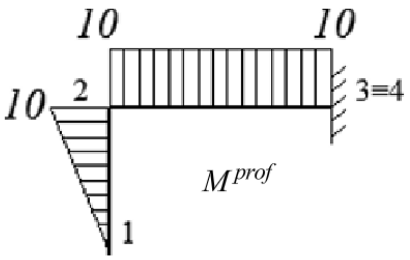
$$N_1 = 0$$

$$N_{2-3}^{H} = 0$$

$$N_{2-3}^{л} = -q * 2 = -6 * 2 = -12\ \text{kH}.$$

$$N_4 = -q * 2 = -6 * 2 = -12\ kH * m$$

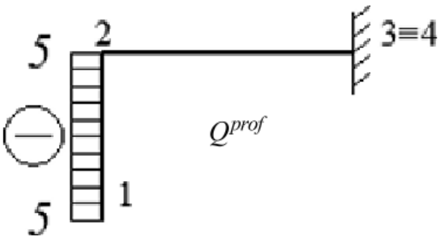
Fig. 2.14 Diagrams of bending moments (a), shear forces (b), and axial forces (c) in the frontal projection.



$$M_1 = 0$$

$$M_2 = -F * 2 = -5 * 2 = -10\ kH * m$$

$$M_{3-4} = -F * 2 = -5 * 2 = -10\ kH * m$$

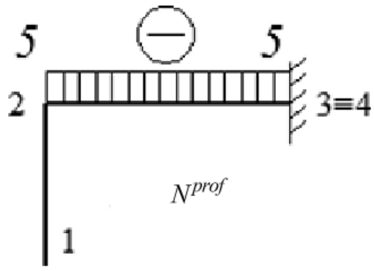


$$Q_1 = -F = -5\ kH$$

$$Q_2^H = -F = -5 = -5\ kH * m$$

$$Q_2^{\Pi} = 0$$

$$Q_{3-4} = 0$$



$$N_1 = N_2^{п} = 0$$

$$N_2^{п} = N_{3-4} = -F = -5\ kH$$

Fig. 2.15 Diagrams of bending moments (a), shear forces (b), and axial forces (c) in the profile projection.

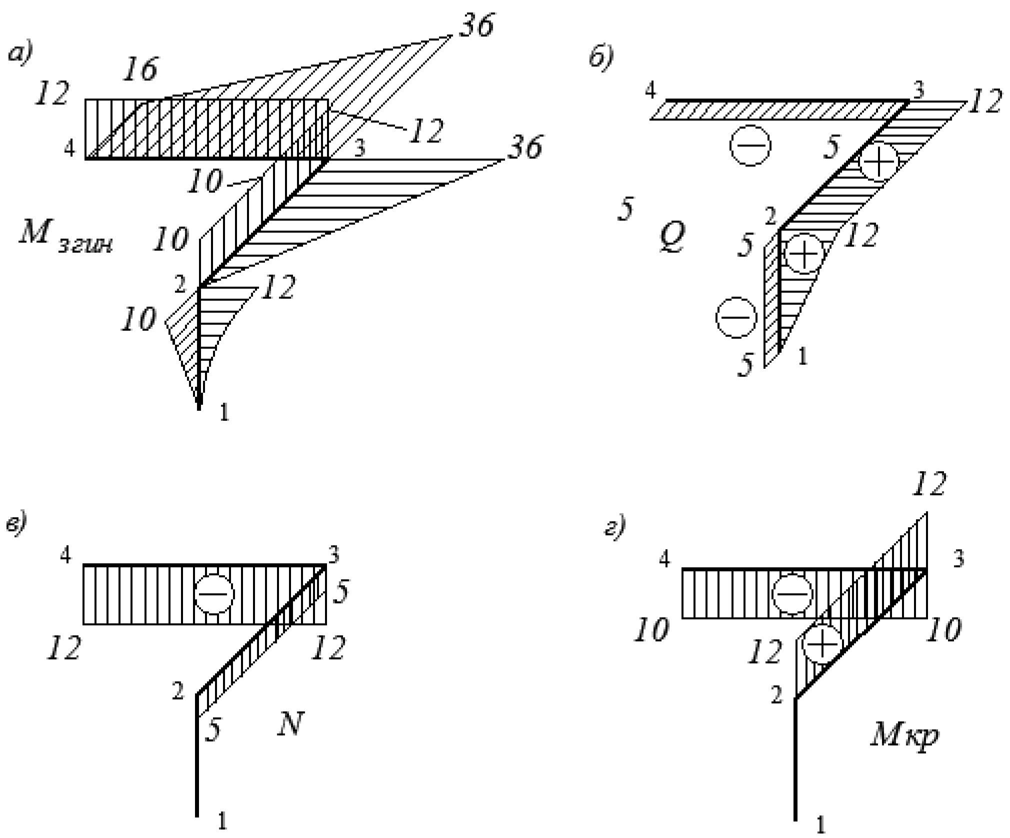


Fig. 2.16 Spatial diagrams of bending moments (a), shear forces (b), axial forces (c), and torsional moments (d)

The torsional moment diagram M_{tor} (Fig. 2.16.d) is constructed on the bars whose numbers coincide with those on the projections of the bending moment diagrams. In this case, the magnitude of the torsional moment is equal to the magnitude of the bending moment at that point and extends along the entire length of the bar.

Horizontal projection 1-2**:** Bending moment $M_{1\text{-}2} = 0 \rightarrow M_{tor1\text{-}2} = 0$;

Frontal projection 2-3: Bending moment $M_{2\text{-}3} = 12$ kN $\rightarrow M_{tor2\text{-}3} = 12$ kN*m;

Profile projection 3-4: Bending moment $M_{3\text{-}4} = 10$ kN $\rightarrow M_{tor3\text{-}4} = -10$ kN*m.

Method 2. When moving from one bar to another, we align the X-axis along the axis of that bar and specify the positive directions of the Y and Z axes accordingly. Then, we adopt the sign convention for forces according to Fig. 2.12. The signs of bending moments can be disregarded; instead, the ordinates are plotted in two planes on the side of the tension fibers on the M diagram.

The diagrams of shear forces, axial forces, and torsional moments are also constructed in two corresponding planes. These diagrams can be oriented arbitrarily, but the ordinates are always plotted normal to the bar axis.

In this system, there is no need to determine the support reactions. We start the construction of the diagrams from bar 3. We determine the result of the action of force F and the uniformly distributed load q on it. These loads induce shear forces and bending moments in the bar, which have the following values:

$$Q_{y3} = -5 \text{ кН}; \ Q_{z3(\max)} = 6 * 2 = 12 \text{ кН};$$

$$M_{z3(\max)} = 5 * 2 = 10 \text{ кНм}; \ M_{y3(\max)} = 6 * 2 * 1 = 12 \text{ кНm}.$$

We proceed to bar 2. The bar is compressed and bent in the vertical plane by force *F*, while the load *q* induces transverse bending in the horizontal plane and torsion of this bar.

$$N_2 = -5 \text{ кН}; \ Q_{y2} = 6 * 2 = 12 \text{ кН}; \ \ Q_{z2} = 0;$$

$$M_{z2(\max)} = 12 * 3 = 36 \text{ кНм}; \ M_{y2} = 5 * 2 = 10 \text{ кНm};$$

$$M_{kp2} = 12 * 1 = 12 \text{ кНm}.$$

In bar 1, force *F* induces transverse bending and torsion, while load *q* compresses and bends it in two planes.

$$N_1 = -6 * 2 = -12 \text{ кН};\ Q_{z1} = 0;\ \ _1 = -5\text{кН};$$

$$M_{z1\,\text{защ}} = 36 - 5 * 4 = 16 \text{ кНm};\ M_{y1} = 6 * 2 = 12 \text{ кНm};$$

$$M_{kp1} = -5 * 2 = -10 \text{ кНm}.$$

We construct the internal force diagrams (Fig. 2.16).

Appendix 2

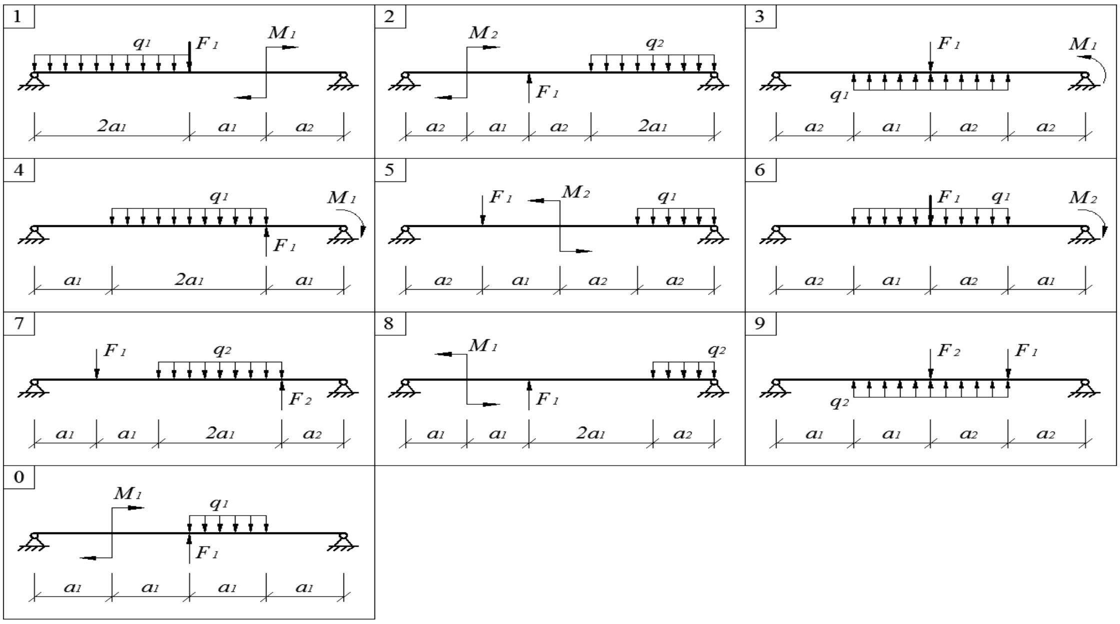


Fig. D.2-1

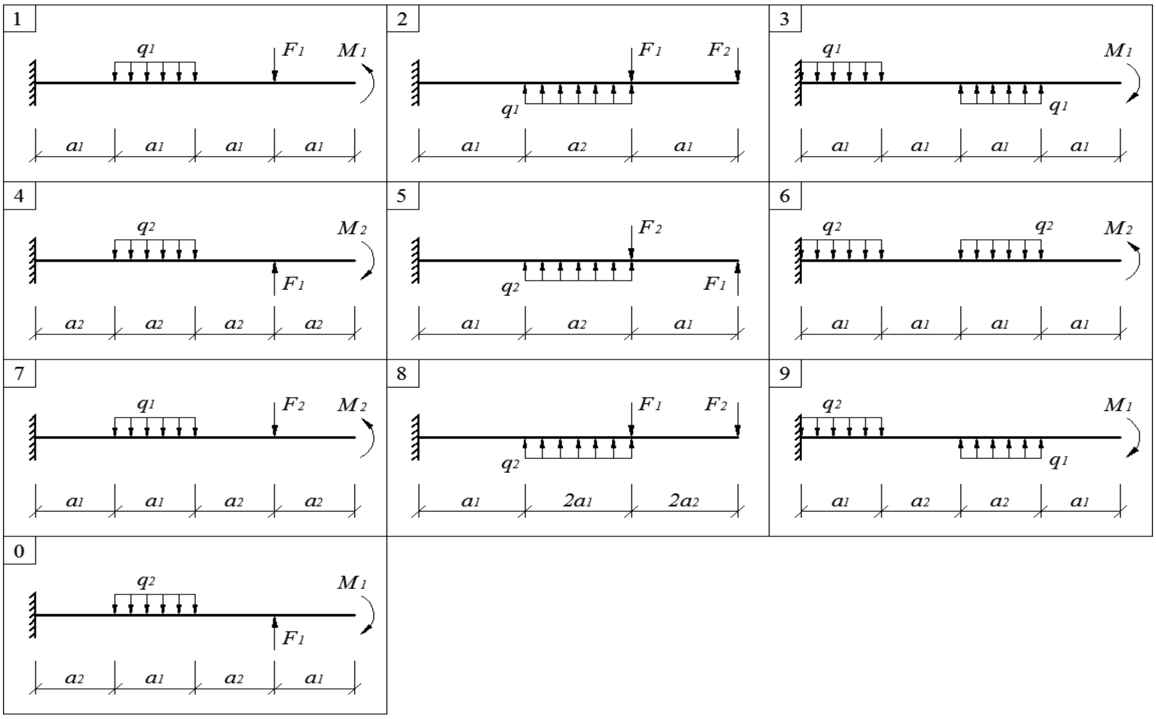


Fig D.2-2

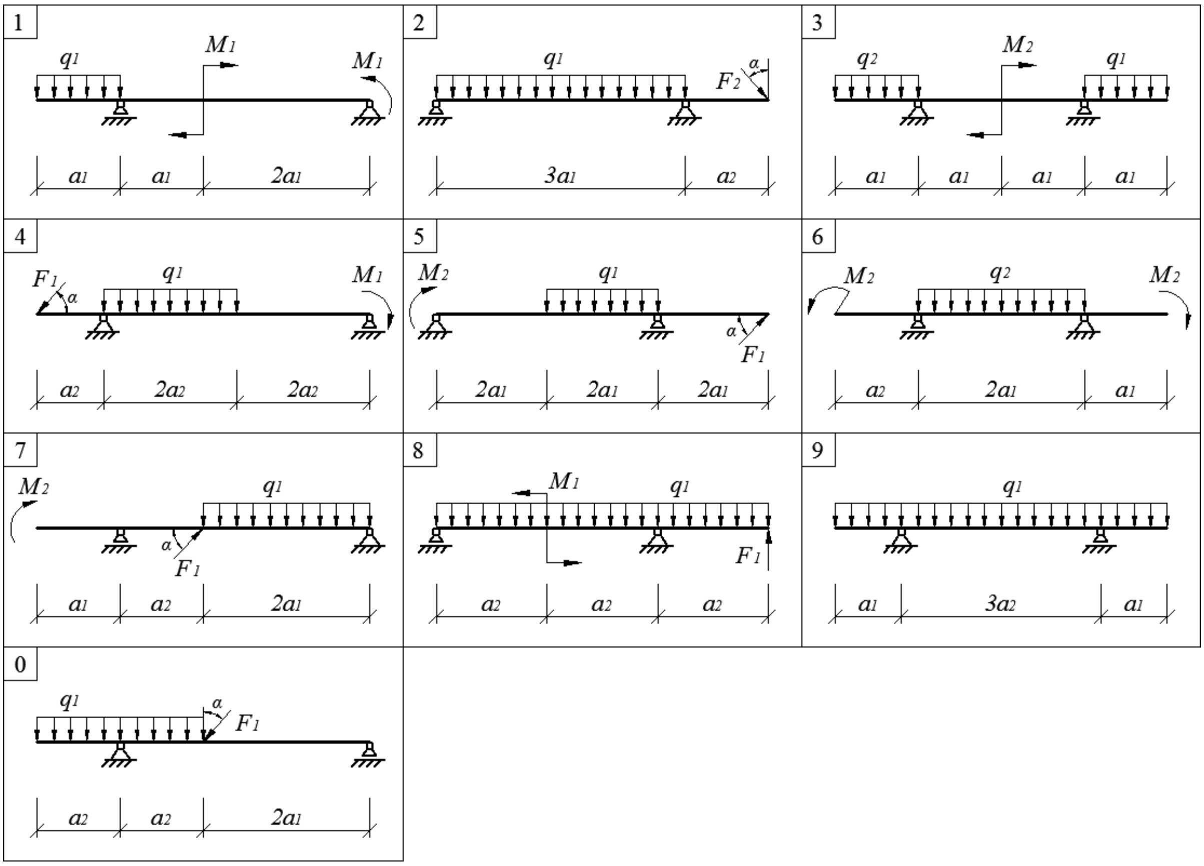


Fig D.2-3

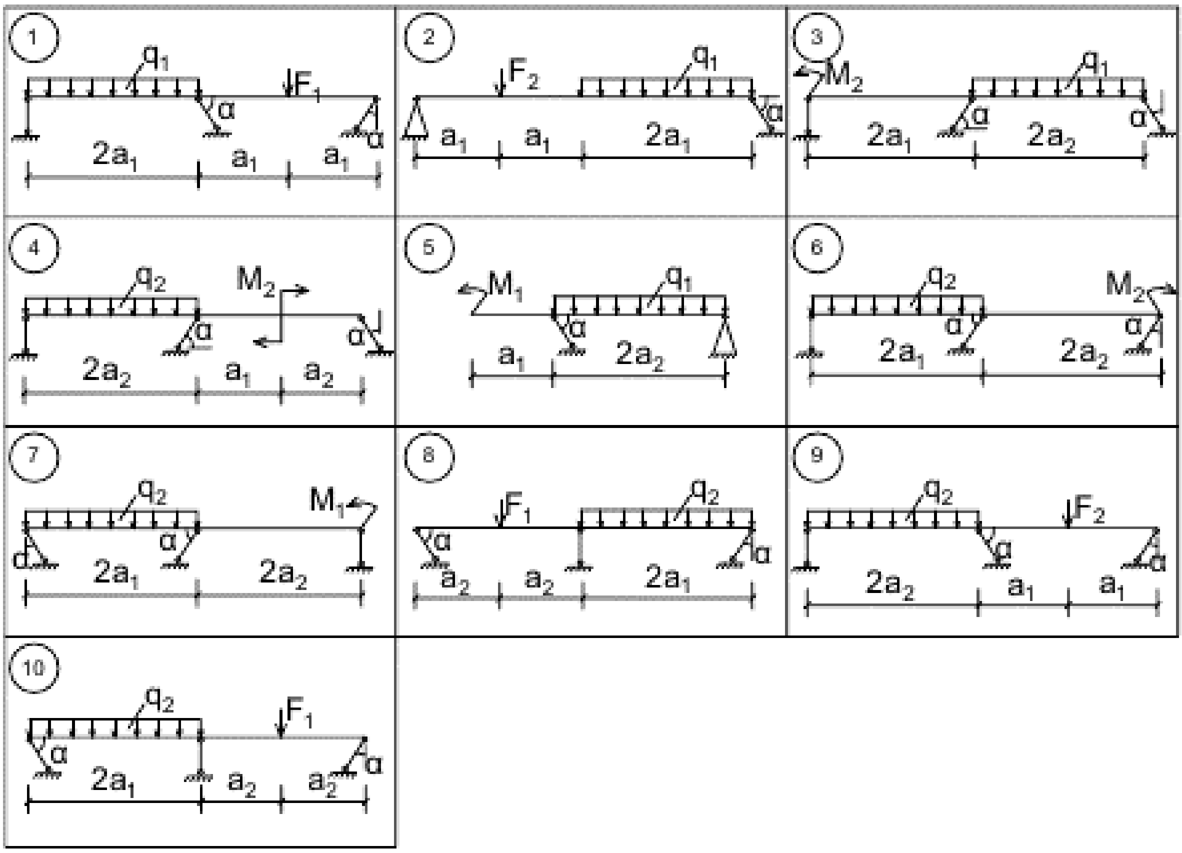


Fig. D.2-4

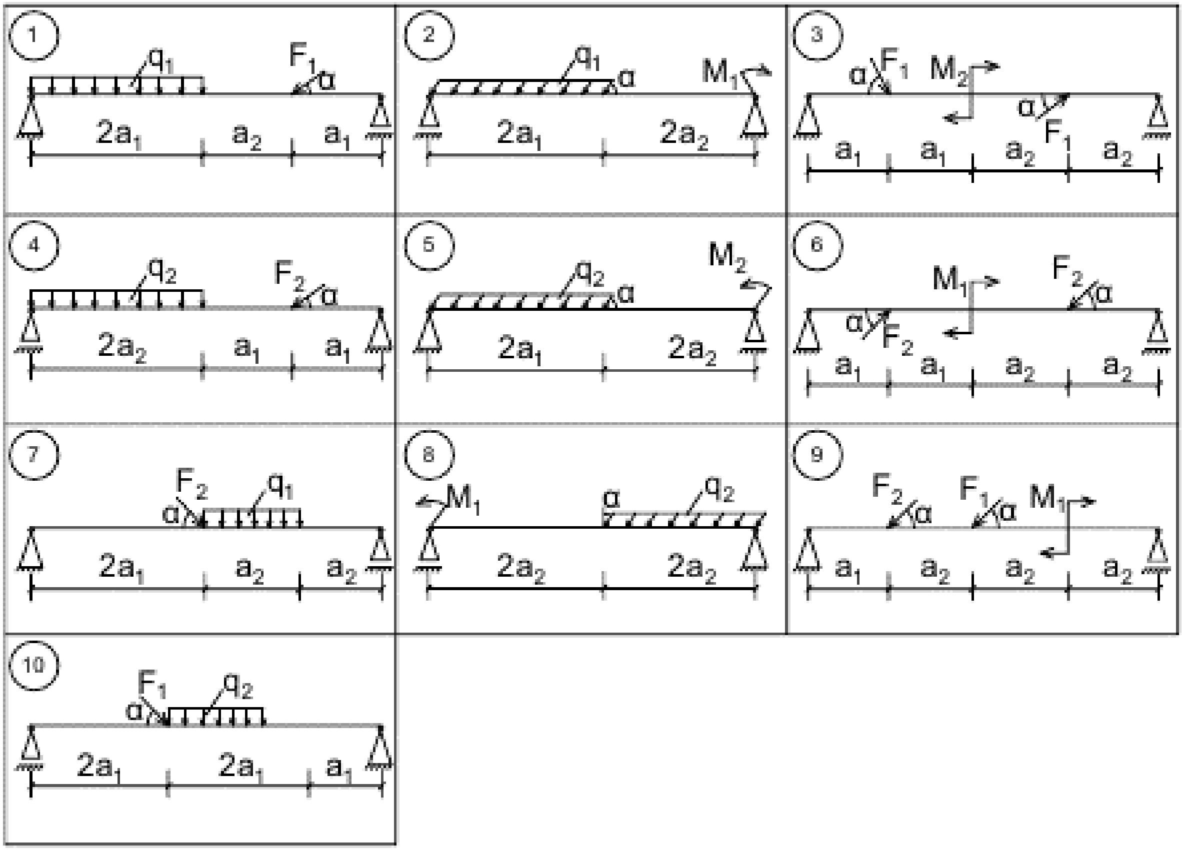


Fig. D.2-5

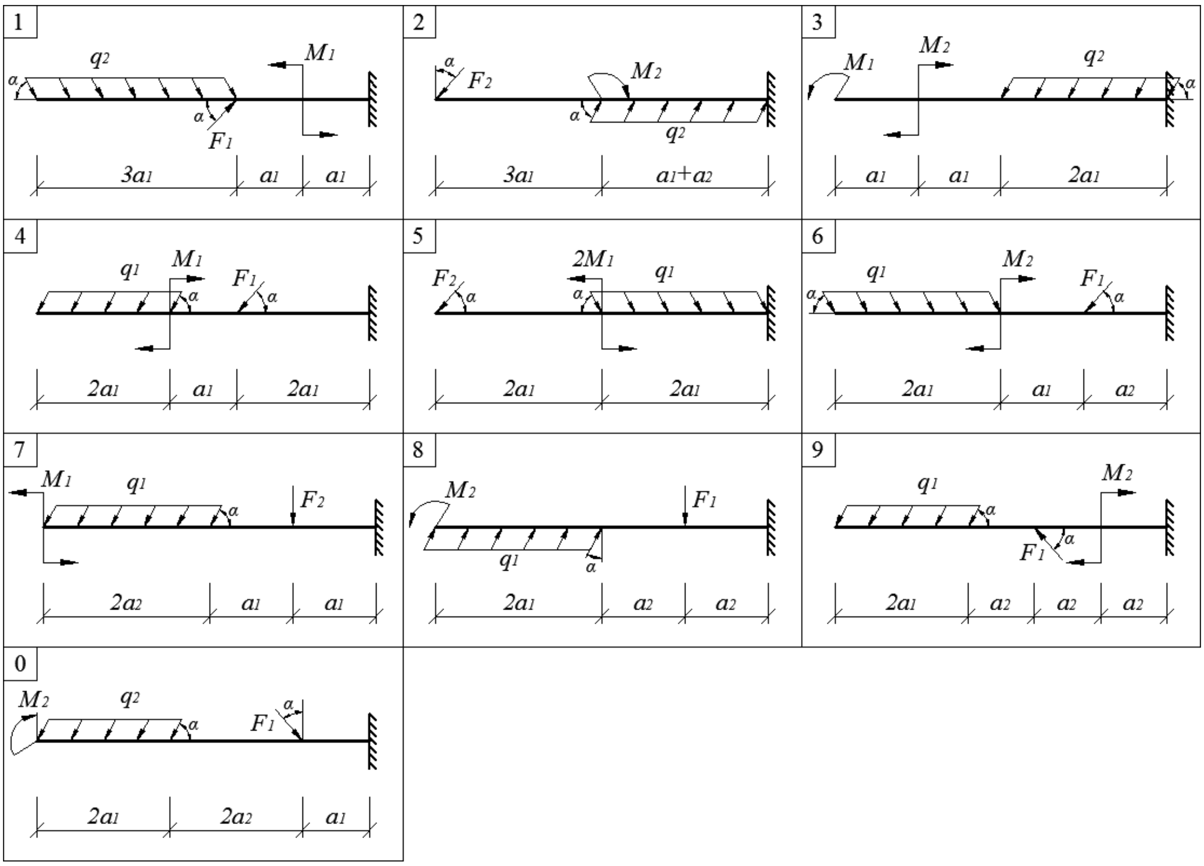


Fig. D.2-6

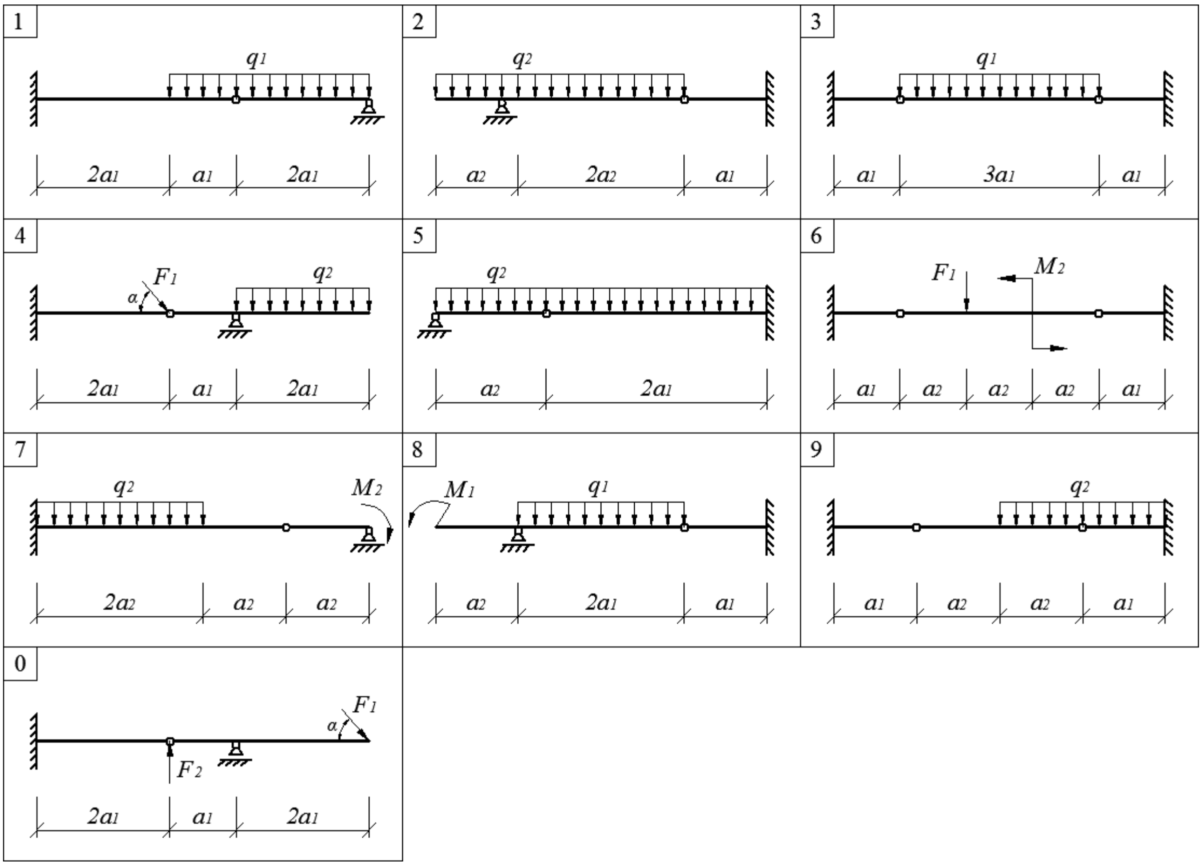


Fig. D.2-7

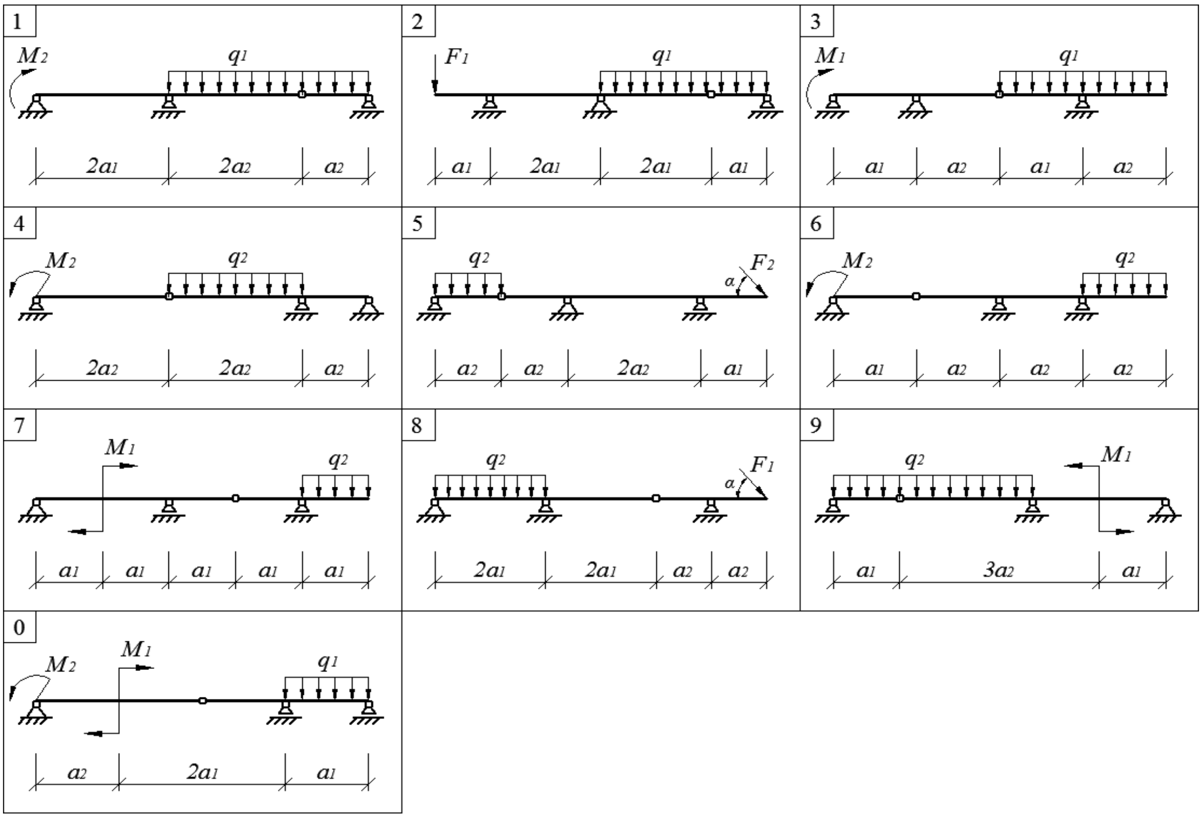


Fig. D.2-8

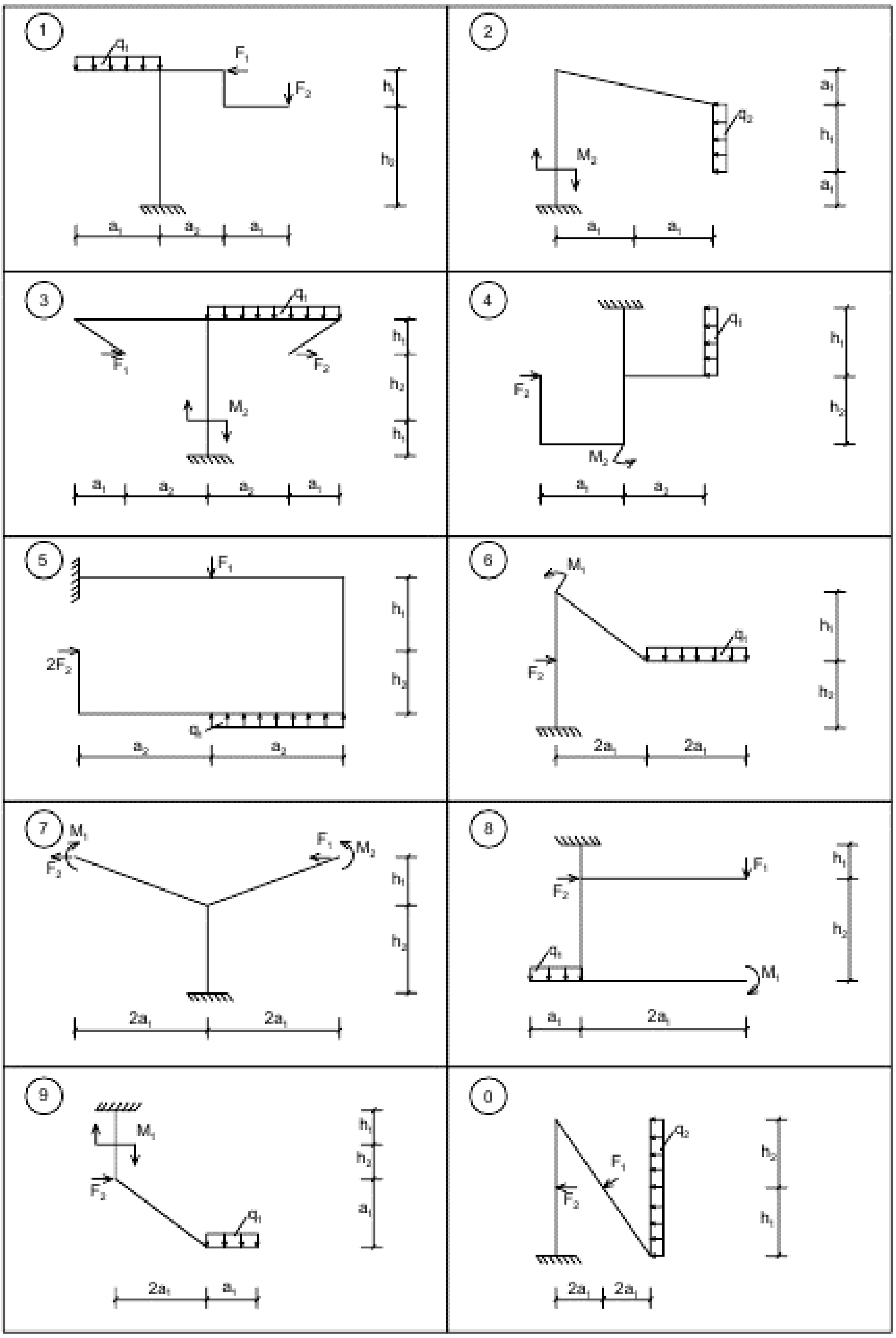


Fig. D.2-9

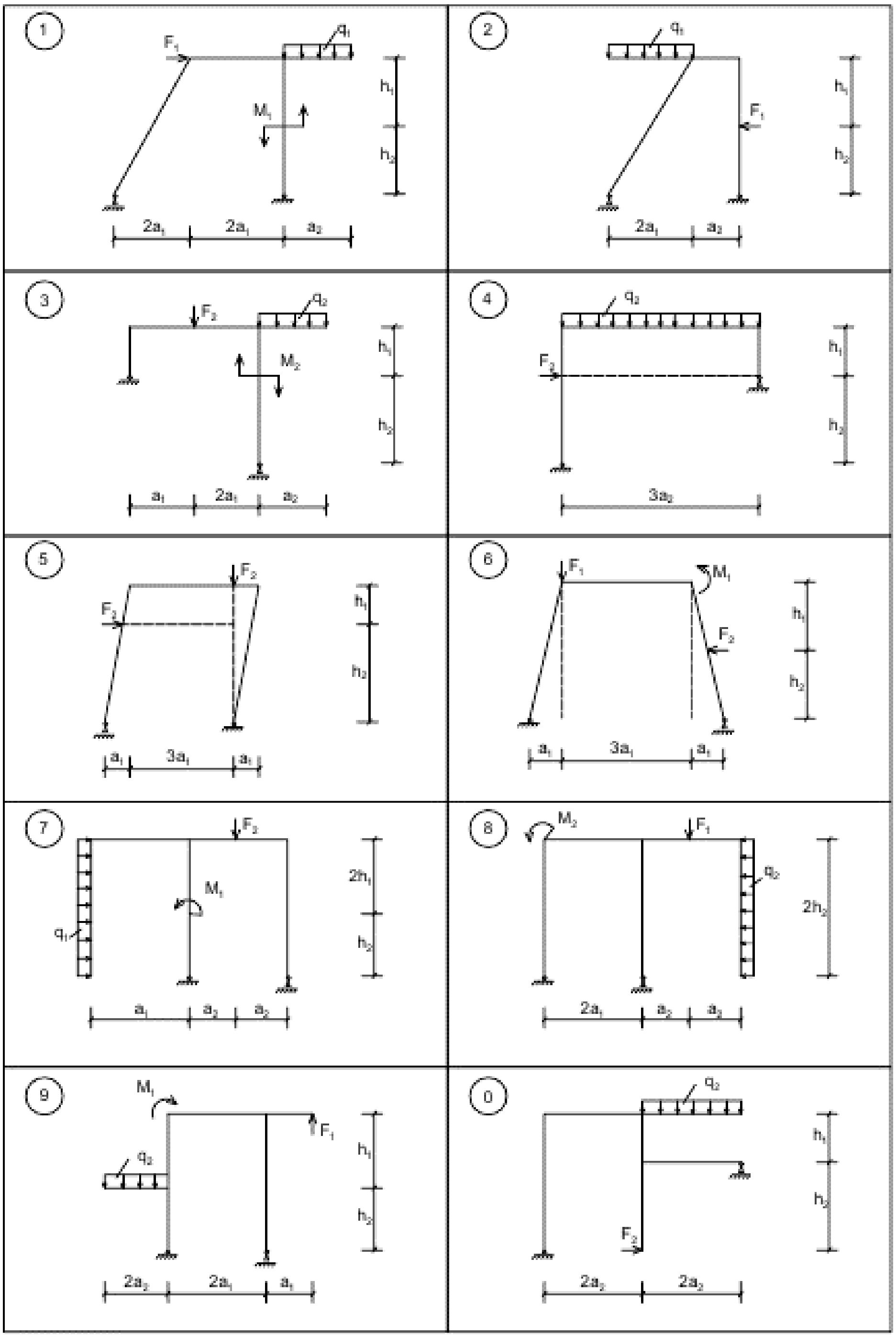


Fig. D.2-10

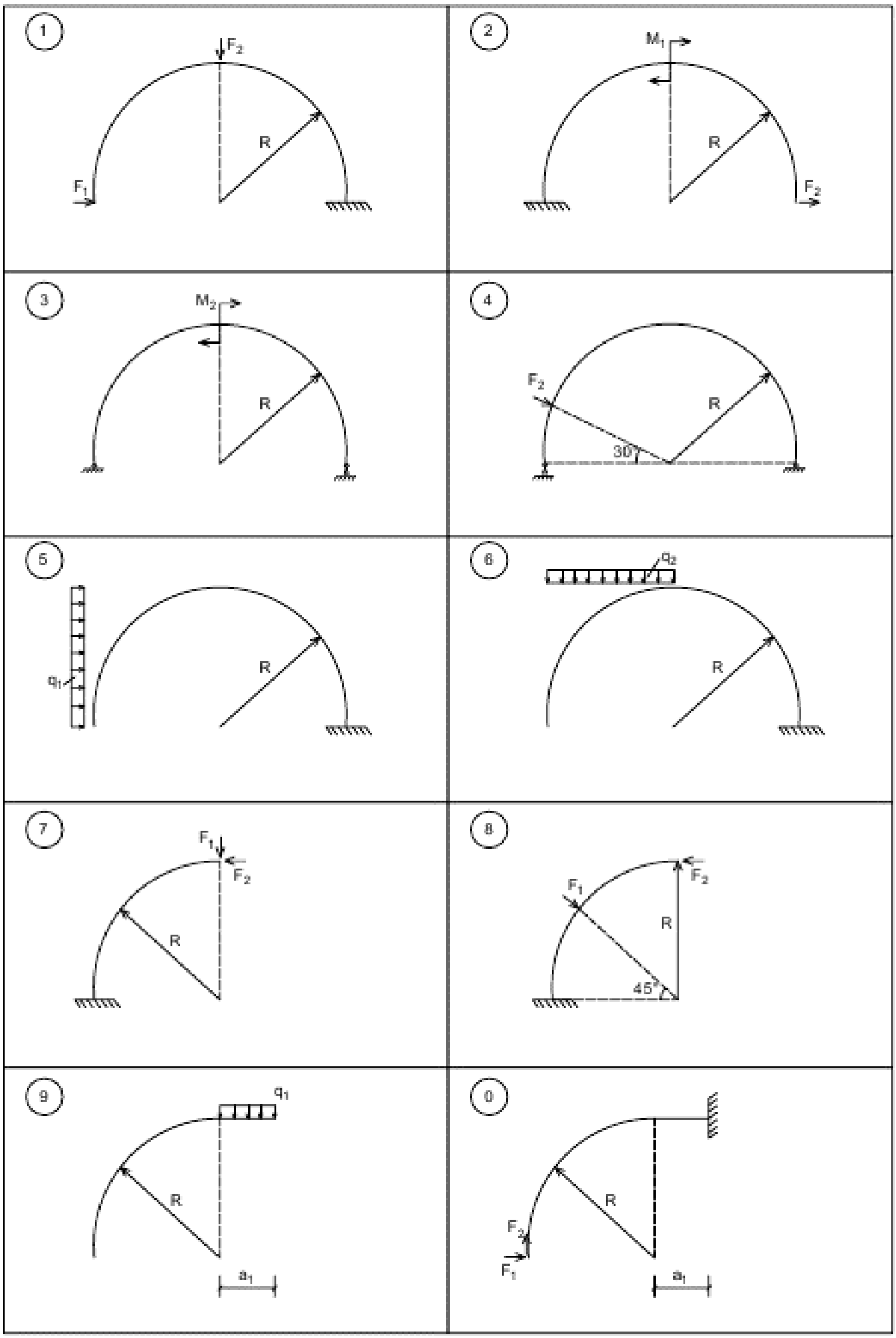

1
F2
R
F1
2
M1
R
F2
3
M2
R
4
F2
R
30°
5
R
q1
6
q2
R
7
F1
F2
R
8
F1
F2
R
45°
9
q1
R
a1
0
R
F2
F1
a1

Fig. D.2-11

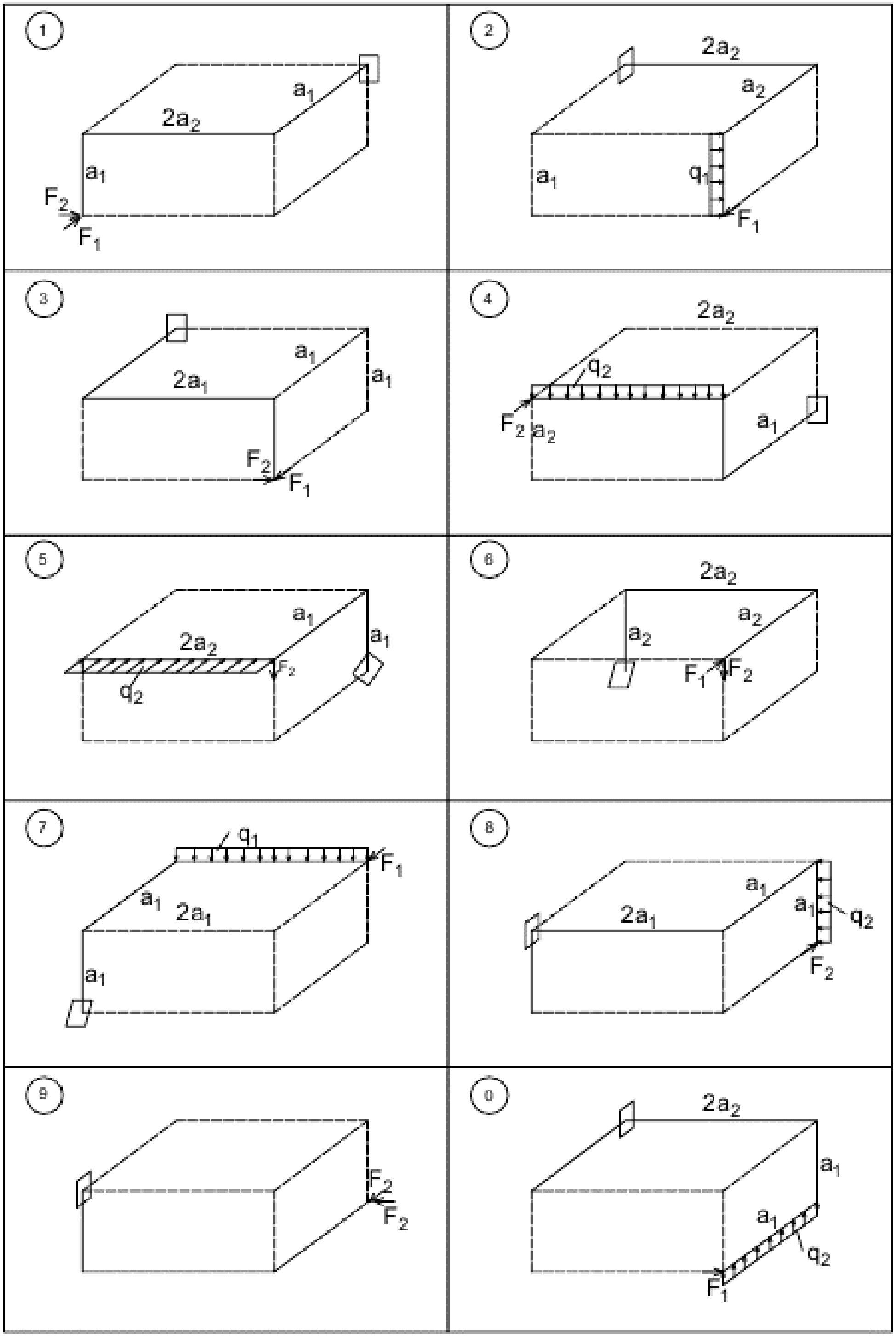

1
a1
2a2
a1
F2
F1
2
2a2
a2
a1
q1
F1
3
a1
2a1
a1
F2
F1
4
2a2
q2
F2
a2
a1
5
a1
2a2
a1
F2
q2
6
2a2
a2
a2
F1
F2
7
q1
F1
a1
2a1
a1
8
a1
2a1
a1
q2
F2
9
F2
F2
0
2a2
a1
a1
q2
F1

Fig. D.2-12

Table D2

Cipher Digit	First Cipher Digit				Second Cipher Digit				Third Cipher Digit			
	a_1	a_2	h_1	h_2	R	M_1	M_2	F_1	F_2	q_1	q_2	α
1	2	3	4	3	6	12	5	10	9	4	3	30°
2	3	4	3	2	5	10	7	8	13	6	5	45°
3	4	2	5	4	4	8	9	14	17	8	7	60°
4	3	3	2	5	5	20	11	20	19	10	9	75°
5	2	4	3	5	6	18	13	24	21	8	7	60°
6	4	3	4	4	3	16	15	16	11	6	5	45°
7	2	2	5	3	4	12	17	18	13	4	3	30°
8	3	2	3	2	5	22	19	10	15	2	7	75°
9	4	3	4	3	6	20	21	18	17	8	9	60°
0	2	4	5	4	4	14	11	22	21	6	5	30°

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Educational and Methodological Publication

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