A Finite Element Technique and Results of Continual Fracture Process Modelling

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Summary. The techniques of modeling of continual fracture process for space circular and prismatic bodies under long-term static or cyclic force loading condition and some results of determination of responsible parts lifetime is presented in this paper. The scalar damage parameter is used to describe the material continual fracture process. A stress-strain problem solution made with semianalytic finite element method (SFEM). It's shown, that the value of residual lifetime could be differ significantly for different loading condition and object configuration.

Key words: long-term loading condition, creep, damage, continual fracture, lifetime, spatial problem, semianalytic finite element method (SFEM).

INTRODUCTION

Structural elements of responsible objects function often under long-term static or cyclic force loading. The process of creep or fatigue, accompanied by the gradual accumulation of scattered damage, the formation and growth of macroscopic defects (fracture zones) are occurs under such a loading conditions. This problem, similarly as well as other aspects of reliability analysis [1, 4], is very important, for a reliable determination of long-term strength and lifetime.

A description of above mentioned processes, which took the name «continual fracture», may be fulfilled efficiently using phenomenological scalar damage parameter, proposed in the works of V.Bolotin, L.Kachanov and Yu. Rabotnov. This approach is developed and implemented for different loading conditions in the publication of M.Bobyr, V.Golub, G.Lvov, Yu.Shevchenko [5, 7, 8, 9, 12] and in publication of foreign scientists (Chen G., Hayhurst D., Lemaitre J., Murakami S., Otevrel I. etc.). However, as noted in [10], the most actual problem is the determining the residual lifetime – time of fracture zone growth after the local loss of the material bearing capacity. Solving of this problem in the spatial definition is not reflected enough in scientific publications. On the other hand, such as highlighted in [11], the residual lifetime value may be up to half of the total time of structure element operation. Thus, it should be taking into account for correct final definition of part's operating time.

PURPOSE OF WORK

The purpose of this paper is to highlight the main provisions of the developed technique for modeling of continual fracture zone growth of spatial bodies and presentation of the results of residual lifetime determination of responsible structural elements under different loading conditions.

INITIAL EQUATION AND METHODS OF ANALYSIS

Continuum fracture mechanics relations. The damage accumulation process described with kinetic equations using phenomenological damage parameter (DP) ω, which changed in time from $\omega(t=0)=\omega_0=0$ to $\omega(t^*)=1$, where: t* – is the time of the local loss of material bearing capacity.

The next view of kinetic equation for DP calculation is most simply for the multi-cyclic force loading [13]:

$$\frac{d\omega}{dN}=A\left(\frac{\sigma}{\sigma_B(1-\omega)}\right)^n, \qquad (1)$$

where: A and n – experimentally determined material constants,
σ_B – tensile strength of the material.

It is expected, that under multi-cyclic loading condition process of material deforms elastically and the loading process can be carried out with variable parameters of the cycle (mean middle stress and amplitude). Therefore, it is provided for construction of DP value determining algorithm, that load process must be divided into a number of steps - steps for problem solving - S^*. Within each stage s (s=1, 2....S^*–1, S^*) load means constant stress σ_{0s} and constant amplitude σ_{as} during the some quantity of cycles N_s. Using this assumption, the DP value by previous load history (up N_S cycles, $N_S=\sum_{s=1}^{S}N_s$) is determined by a formula which obtained in [13] as a closed form solution of equation (1):

$$\omega_S=1-\sqrt[(n+1)]{1-\frac{A}{(n+1)\sigma_B^n}\sum_{s=1}^{S}(\sigma_{as})^n N_s}\ . \qquad (2)$$

A DP value description under long-term static loading condition (when a creep process presence) conducted using the follow expression [7]:

$$\frac{d\omega}{dt}=C\left[\frac{\sigma_e}{1-\omega^r}\right]^m\frac{1}{(1-\omega)^q}\omega^\beta, \qquad (3)$$

where: C, m, q, r, β – experimentally determined material constants, which are functions of temperature, σ_e – equivalent stresses calculated according to the chosen strength criterion.

Semianalytic finite element method (SFEM). The solution of evolutionary problems of spatial bodies deformation process requires significant computational expences and special algorithms for regarding of damaged accumulation process and fracture zones growth simulation. It is not always possible to solve these problems using modern powerful finite element software systems (ANSYS, ABAQUIS, etc.), based on traditional three-dimensional finite element problem definition.

SFEM is an effective instrument for numerical modeling of stress-strain state and deformation process of canonical form spatial bodies – inhomogeneous circle and prismatic bodies. The term "inhomogeneous" is used in the sense of the variability of the physical, mechanical properties and geometrical dimensions of the body along the forming. Being based SFEM, a discrete calculation model suggests the finite element mesh in the cross section of the examined object, and one finite element (FE) to be used in the orthogonal towards the cross sectional plane (along the forming, i.e. $z^{3'}$ coordinates. Thus, the FE size in the $z^{3'}$ direction is the same as the body one (Fig. 1).

SFEM allows significantly reduce the computational expenses for solving of spatial problem, particularly on the stages of stiffness matrix calculating and FEM linear equations systems solving. The efficiency and accuracy of the method is shown for a wide range of linear and nonlinear problems of mechanics [2-4], where readers can also find a more detailed description of the method features, its implementation and links to additional author’s publications.

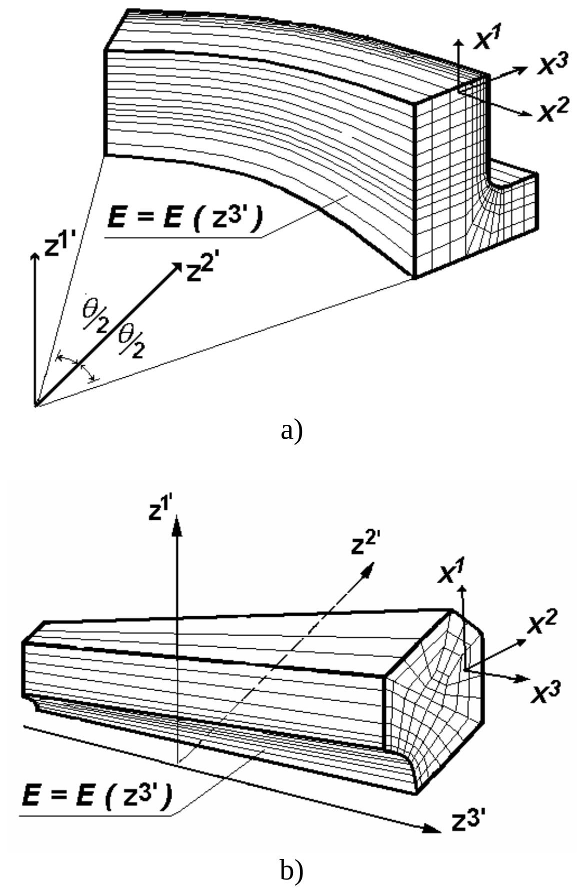


Fig. 1. Circle (a) and prismatic (b) inhomogeneous body

FINITE ELEMENT ALGORITHMS FOR CONTINUAL FRACTURE PROBLEM SOLUTION

A problem of stress-strain state parameters determination under linear and nonlinear deformation process performed by the algorithm based on the use of the implicit integration over the time scheme with help of Newton-Kantorovich iterative procedure:

$$\{\Delta U_l\}_n^m = \{\Delta U_l\}_{n-1}^m + \beta\,[K_{ll}]^{-1}\left(\{Q_l\}^m - \{R_l\}_n^m\right),$$

where: is $1 \le \beta < 2$ – relaxation parameter, $\{Q\}^m$ – vector of full loads in nodes on step m, $[K_{ll}]$ – FE stiffness matrix, $\{R_l\}_n^m$ – vector of nodes reactions on iteration n of step m.

Creep problem solution considering damage accumulation is being executed by means of step-by-step algorithm on the parameter of time. When starting each iteration n of a step m, stress values σ_{ij} are calculated considering creep deformation process by the formula:

$$\left(\sigma_{ij}\right)_n^m = \frac{1}{3}\delta^{ij}\left(\overline{\sigma_{ij}}\right)_n^m + \left(s^{ij}\right)_n^m. \qquad (4)$$

Components of stress tensor $\overline{\sigma_{ij}}$ are defined in compliance with the Hook's law considering an increment of total deformation:

$$\left(\overline{\sigma_{ij}}\right)_n^m = \left(\sigma_{ij}\right)_{n-1}^m + \left(\overline{\Delta\sigma_{ij}}\right)_n^m,$$

while the components of stress deviator $\left(s^{ij}\right)_n^m$ relate to an increment in creep deformation $\Delta\varepsilon_{ij}^c$:

$$\left(\Delta\varepsilon_{ij}^c\right)_n^m = \left(\xi_{ij}^c\right)_n^m \Delta\, t_m ,$$

$$\left(s^{ij}\right)_n^m = \left(\overline{s^{ij}}\right)_n^m - G_1\left(\Delta\varepsilon_{ij}^c\right)_n^m , \qquad (5)$$

where: $\left(\xi_{ij}^c\right)_n^m = \dfrac{3}{2}\left[\xi_i^c\right]_m^n \dfrac{\left(s_{ij}\right)_n^m}{\left(\sigma_i\right)_m^n}$ – components of creep deformation rate tensor, $\xi_i^c = \dfrac{d\varepsilon}{dt}$, $G_1 = E\,/\,(1-2\mu)$ – elastic constants, Δt_m – time interval value.

The DP values addition $(\Delta\omega)_m$ and accumulated DP values ω_m on a time interval m calculated with next relation**:**

$$\omega_m = \omega_{m-1} + \left(\Delta\omega\right)_m = \omega_{m-1} + \left(\frac{d\omega}{d\,t}\right)_m \Delta t_m . \quad (6)$$

The criterion of local loss of the material bearing capacity is $\omega\,(t^*) > \omega^*$, where $\omega^* \approx 1$ – critical DP value. It's fulfillment in the some point of studied object K with coordinates $\left(z^{i'}\right)_K = z^{i'*} = \left\{z^{1'*},\ z^{2'*},\ z^{3'*}\right\}$, indicates the transition of the scattered damages accumulation process, which accounted integrally using DP, to occurrence of macroscopic defects – initial areas of continual fracture. This points in time determining the value of the estimated lifetime of studied object.

Continual fracture zone growth modeling. To simulate the initial macroscopic defects occurrence at the point *K* the area with volume V_0 introduced there at the time point $t = t^* + \Delta t$ (Fig. 2, a). The size of this area in plane $z^{1'} - z^{2'}$ is the same of FE size, in which condition $\omega > \omega^*$ reached. The size $\Delta z^{3'}$ in the $z^{3'}$ direction defined as the sum of half the distance from the point *K* to neighboring integration points in FE (named as *K-1* and *K+1*, $\Delta z^{3'} = a_{k-1}/2 + a_k/2$, Fig. 2, c). Values of stress and elastic modulus of the material taken as being equal to zero within a specified area:

$$\sigma_{ij}(t = t^* + \Delta t,\ \ z_i = z_i^*) = 0,$$

$$E(z_i = z_i^*) = 0\,. \qquad (7)$$

Volume V_0, the value of which is caused by the discrete model parameters, defines the minimum increment of the characteristic size of the fracture zone in the course of its growth.

Implementation of (7) is carried out using special FE with adjusted values of physical and mechanical constants. The stress-strain state parameters and DP values determine by (3) - (6) during following point of time. This is accompanied by a gradual increase of fracture zone by acceding to it of a new volume V_m at time intervals t_m after fulfillment of condition $\omega > \omega^*$ in appropriate points. The procedure of continual fracture zone growth modeling is performed to achieve a zone of critical size (volume) V^* (Fig.2, b). The appropriate time interval (number of cycles) determines the residual lifetime (vitality) of the object after the of fracture zones occurrence.

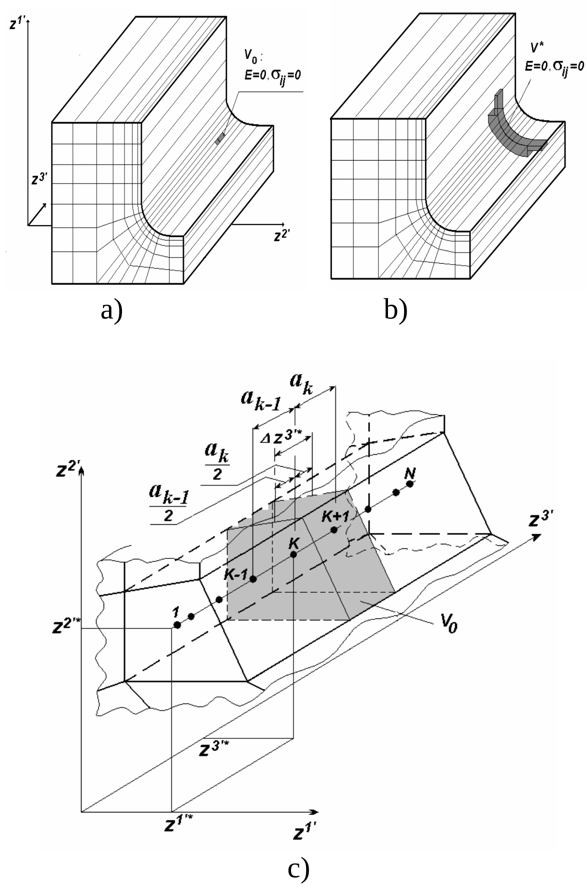


Fig. 2. Stages of continual fracture zone growth (a,b) and special finite element (c)

RESULTS OF FINITE ELEMENT MODELING OF CONTINUAL FRACTURE ZONE GROWTH AND RESIDUAL LIFETIME DEFINITION

The allowed approaches being spent to solving of practical problems of residual lifetime definition of responsible structure element – the connecting union under multi-cyclic loading condition and the blade of a gas turbine under creep.

The connecting union (choke) of valve settings for high pressure polyethylene synthesis is a massive circle (cylindrical) body loaded with cyclic internal pressure. The initial defect is available on the inner surface of the choke - an weakened area of degraded material physical and mechanical properties. General view of the object and the FE discrete model, used for solving and describing the defect presence, shown in Fig.3. The initial elastic stresses distribution in the absence of defect is uniform on choke's height ($z^{1'}$ axis).and variable along the it's radius.

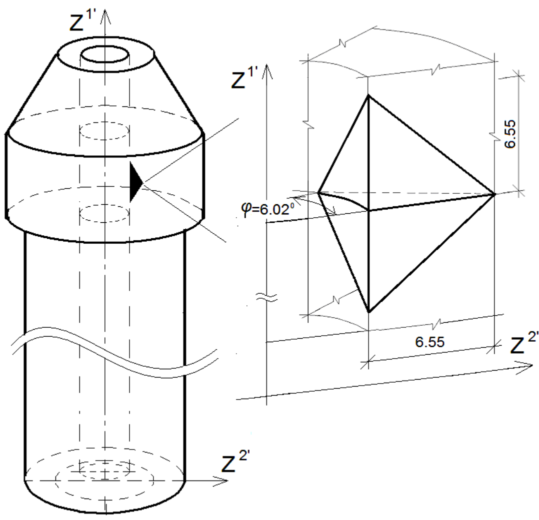


Fig. 3. The connecting union with initial defect

The description of the damage accumulation process performed with eq.(2) using $\sigma_B = 1300$ MPa, $A = 1.5495 * 10^{-2}$ and $n = 4{,}267$. The degradation of material mechanical properties within the defect area performed with linearly change of n in the range 4,267-4,4 (increase of the value n indicates more intensive damage accumulation at the same level of stress). Estimated lifetime of the choke (until the local loss of the material bearing capacity) in the absence of the defect was $4.9 * 10^9$ cycles, and $N* = 3.89*10^9$ for defect presence. The obtained DP values rapidly decreases at choke wall thickness with distance from the inner surface and at the distance of 3 mm from it DP value is less then 0,1. Thus, after the local loss of the material bearing capacity on the inner surface the wall remains virtually intact and the choke may be used stay in operation.

The simulation of fracture zone growth was conducted in axisimmetrical and spatial statement using FEM model, shown on Fig.4. It is shown, that time for zone growth in the radial direction (of the wall thickness) to depth 1-5 mm, which obtained in axisymmetrical statement, is half less than one obtained in spatial statement. This difference gradually decreases: at a zone depth 12 mm (corresponding to half of the wall thickness) it is only near 5%. But in any case, the magnitude of the residual life after the fracture zone growth is almost an order greater than the time to zone occurred. Wherein, it should be noted, that during all of time of the fracture zone growth the detail keeps tight and relevant performance properties (Fig. 5).

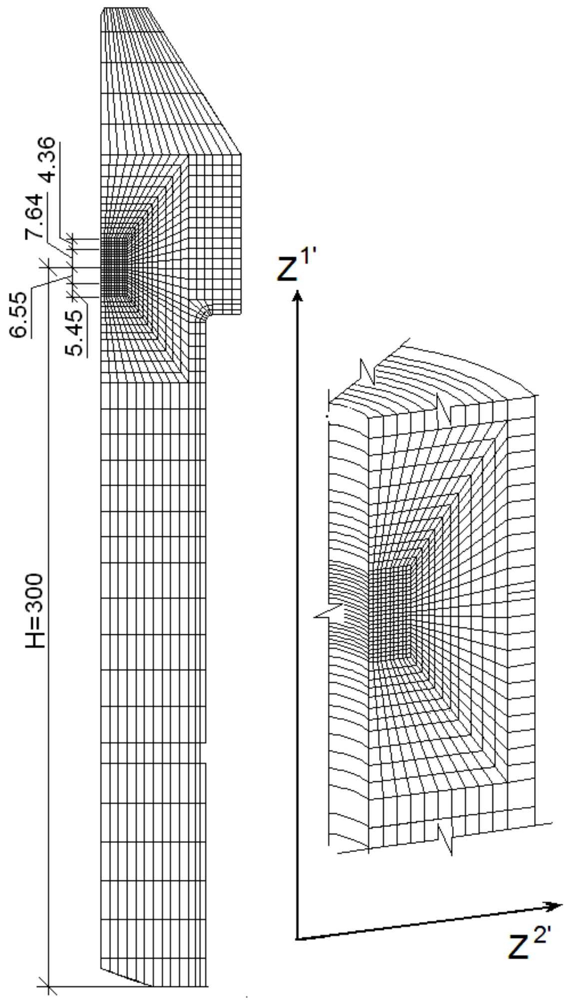


Fig. 4. SFEM discreet model of connecting union pipe with defects

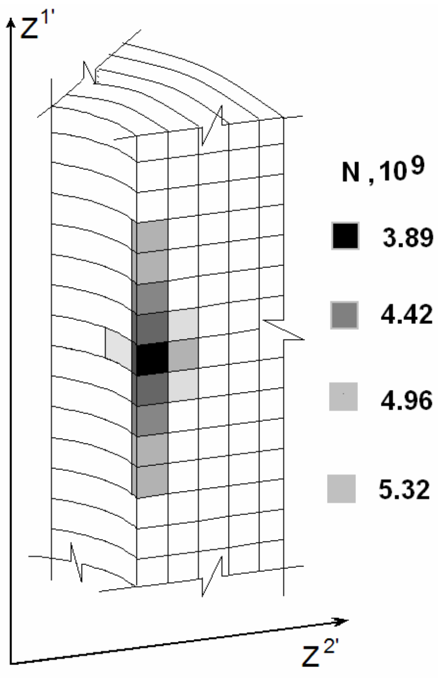
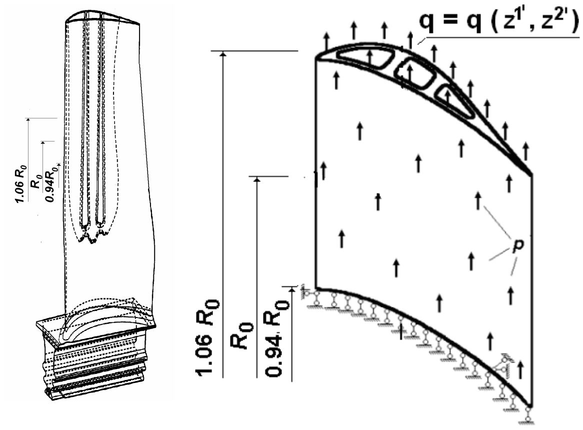


Fig. 5. Configuration of continual fracture zone after different number of cycles (b)

Stationary gas turbine blade is a spatial body of complicated shape. The blade is swirled at the vertical axis, has a variable at height cross-sectional area. It is influenced by centrifugal forces in a heterogeneous, both in height and in cross-section, temperature field (Fig.6, a). Based on results of elastic deformation modeling of blade, based on three-dimensional FEM, a dangerous cross section R_0 was chosen. This section characterized with combination of average strain σ_0 and average temperature T_0 , which leads to the most intensive creep process and damage accumulation. Listed values (σ_0 and T_0) are used further to describe the design scheme and the results of the problem solving. Creep deformation process modeling is made for a blade fragment with size 0,94 $R_0 < R <$ 1,06 R_0. Fragment is loaded with its own centrifugal load *p*. The simulation of the upper part of blade in section R = 1,06 R_0 implemented with unevenly distributed load $q = q(z^{1'}, z^{2'})$ that meets stress values, been applicated in this cross-section (Fig. 6, b).



a) b)

Fig. 6. Gas turbine blade: general view (a) and design scheme of blade fragment(b)

The description of creep and damage accumulation processes is carried out with the following equation:

$$\frac{d\varepsilon_c}{dt} = \frac{B\sigma^n}{(1-\omega)^r}, \quad \frac{d\omega}{dt} = C\left(\frac{\sigma}{1-\omega}\right)^m \frac{1}{(1-\omega)^q}, \quad (8)$$

where: *B = B(T), C = C(T), m = m(T), n = n(T), r = r(T), q = q(σ, T)* – material constants, *T* – temperature.

The location of initial fracture zone within the fragment corresponding to the maximum value damage parameter was determined due to modeling of blade deformation under creep. Its location is aligned with the zone of maximum values of the DP at height of blade fragment (Fig. 7). It was required to use a finite element models with a significant number of nodes in a cross-section to determine of fracture zone's size and shape in the process of it's growth. The results fracture zone growth modeling up to complete loss of bearing capacity showed, that the proposed facility value of residual lifetime is low and is about 5% of the estimated lifetime. This suggests that the actual value of the blade lifetime is determined by local loss of bearing capacity in fact (Fig. 8).

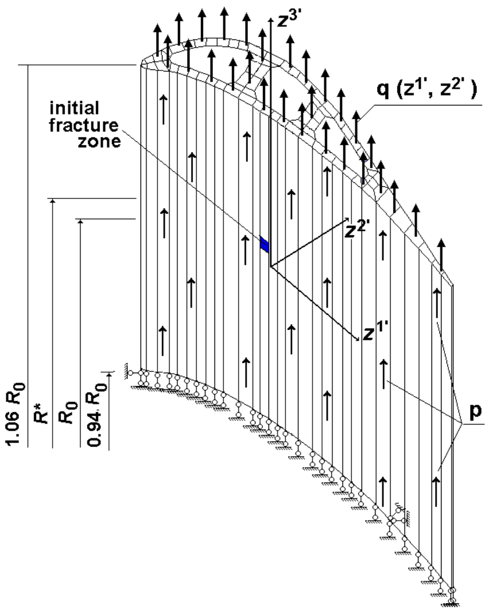


Fig. 7. Base SFEM discreet model and initial fracture zone (c)

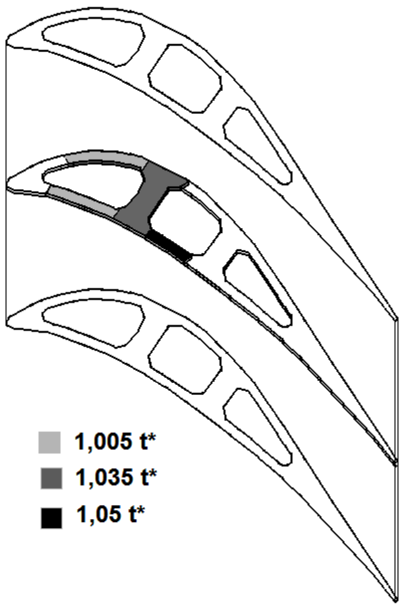


Fig. 8. Continual fracture zone configuration in blade fragment

CONCLUSIONS

1. Developed in this paper methods for modeling of continual fracture process and fracture zone growth allows to determine of estimated and residual lifetime values for responsible structural elements that work under long-term static and high-cycle loading condition.

2. The values of residual lifetime may differ significantly for different objects and loading conditions. Thus, it's need to study a problem of extending of details operation time after local loss of bearing capacity in each individual case.

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КОНЕЧНОЭЛЕМЕНТНАЯ МЕТОДИКА И РЕЗУЛЬТАТЫ МОДЕЛИРОВАНИЯ ПРОЦЕССОВ КОНТИНУАЛЬНОГО РАЗРУШЕНИЯ

Аннотация. В статье представлена методика конечноэлементного моделирования процессов континуального разрушения пространственных тел вращения и призматических тел под действием длительного статического и циклического нагружения. Для описания континуального разрушения использован скалярный параметр повреждаемости материала. Определение напряженно-деформированного состояния выполняется на основе полуаналитического метода конечных элементов. Показано, что относительная величина дополнительного ресурса существенно зависит от условий нагружения и конфигурации объекта.

Ключевые слова: длительное нагружение, ползучесть, повреждаемость, континуальное разрушение, ресурс, пространственная задача, полуаналитический метод конечных элементов (ПМКЭ).