UDC 539.3:004.94: 624.074

SOME ASPECTS OF MODELLING NONLINEAR BEHAVIOUR OF REINFORCED CONCRETE

M. Barabash[1,2],
Doctor of Technical Sciences, Senior Research Officer,
Professor of Computers Technologies Department

[1]*LIRA SAPR Ltd,*
7a, Kiyanovsky side street (provulok), Kyiv, 04053, Ukraine
[2]*National Aviation University,*
1,Kosmonavta Komarova, 03058, Kyiv, Ukraine

Abstract. The paper deals with some aspects of modelling the structures' behaviour: the 'engineering nonlinearity' method; determining stresses on the basis of nonlinear dependences 'stress-strain'. The 'engineering nonlinearity' method enables you to indirectly consider physical nonlinearity while computing by the standard method. The 'engineering nonlinearity' method enables you to consider physically nonlinear behavior of reinforced concrete section by iteration and step-type method. The method makes it possible to determine the stiffness parameters of the section. These parameters may be reduced because of the crack propagation, plastic strain in concrete and reinforcement. Concept of the method is described. Suggested method 'engineering nonlinearity' enables the user to consider stiffness distribution more accurately. This method is almost similar to standard methods of linear analysis, that is, it is possible to carry out analysis on all types of loads, compose DCF and DCL, analyse reinforcement. Comparison study for peculiar features of 'Engineering nonlinearity' method is performed. Analysis results for the test problem (based on engineering nonlinearity) are provided. Analysis results for the test problem shows some redistribution of forces and convergence of results obtained in 'engineering nonlinearity' method and in analysis with account of physical nonlinearity. This approach makes it possible to use the 'engineering nonlinearity' method for computing and modelling the erection process, analysis of panel buildings (platform joints), etc. The 'Cross-section Design Toolkit' module supports nonlinear analysis for a certain set of forces. The proposed methods for modelling and analysis of structures with account of their life cycle enable us to find out dangerous tendencies at the design, erection and further maintenance stage of the structure and to prevent the possible destruction both for separate structural elements and the object as a whole.
Keywords: physical nonlinearity, engineering nonlinearity, life cycle, design model, computer modelling

Introduction. New methods of mathematical physics (for solving problems of dynamics, stability, physical and geometric nonlinearity) were developed due to unlimited capabilities for computer models on the basis of FEM. For example, solving physically nonlinear problems within the limits of the active load (regardless of a number of simplified hypotheses) provides the engineer with much more information for estimating the stress-strain state compared with design models based on elastic-linear assumptions.

We can suppose that the independent role of design models in linearly elastic statement will subsequently be rapidly decreasing and they will be given an auxiliary role to solve nonlinear problems on the basis of linearization methods.

A great contribution to the general theory of numerical analysis methods, including the creation and development of finite element method, methods of nonlinear analysis of structure was made by V.A. Bazhenov [1, 3], Yu.V. Veryuzhsky, A.S. Gorodetsky, V.I. Gulyaev, V.N. Kislooky [2], A.V. Perelmuter, A.S. Sakharov [2], V.I. Slivker, V.K. Tsykhanovsky and others.

Many works have been devoted to the methods of solving nonlinear problems in structural mechanics [1-3, 5].

To approximate design models generated at the design stage to the actual conditions of their maintenance, it is necessary to consider the change in the stress-strain state at each stage of the life cycle of the building object [6]. Development of methods of computer modelling is of high importance. These methods are directed to maximize the use of load-bearing capacity of structures while reducing the material consumption and ensuring structural safety.

Along with the standard approaches for determining the stress-strain state of the structure (fixed load – design model – stress-strain state), the modelling of structures' behaviour becomes more important. Behaviour of structures is examined in various situations with account of various features and factors.

Let us consider some aspects of modelling behaviour of structures.

1. Concept of the method 'engineering nonlinearity'. The method makes it possible to determine the stiffness parameters of the section. These parameters may be reduced because of the crack propagation, plastic strain in concrete and reinforcement. Creep, cracks and other specific features of reinforced concrete cause a change in the stiffness parameters of elements at the early stages of loading, including the maintenance stage. This causes redistribution of forces, significant increase in displacements compared with linearly elastic analysis. Account of these factors is regulated by normative documents. Thus, in the normative documentation of some countries it is pointed out that in practical calculations it is possible to take into account lowered stiffness values – for tensioned elements, use a reduction factor of 0.3, for compressed ones - 0.6. However, in reality there are no elements only bent or compressed. As a rule, the elements are eccentrically compressed or eccentrically tensioned. In [4] it is suggested to take into account the reduced stiffness by the method of integral moduli of elasticity. The method introduces single modulus of elasticity for the whole section. Bondarenko indicates that normal stresses vary along the height of the element. Nonlinearity of the strain of material predetermines the difference in the moduli of elasticity at points with different stresses.

These features of behaviour of reinforced concrete are taken into account more precisely by the step-type method [5]. Analysis of structure with account of physical nonlinearity (that is regulated by normative documents in the strict mathematical sense of this process when used in general engineering calculations) has a number of factors that are not acceptable in the case of general calculations of complex structural systems:

- such analysis may be carried out only for one load and it cannot be used in DCF or DCL;
- defining input data is time-consuming process;
- excessive resources due to multiple solution of linearized equations, it causes considerable increase in analysis time for complex structural systems.

These factors cause creation and development of the 'Engineering Nonlinearity' method. On the one hand, this method indirectly takes into account the reduced stiffness due to physical nonlinearity; on the other hand, it allows the engineer to use the standard analysis method [5, 6, 7].

The concept of the method is as follows.

Analysis of structures with account of engineering nonlinearity allows us to determine how the forces are redistributed in the elements, how displacements are increased and many other features of the structure's behavior that cannot be determined with linear elastic calculation. In fact, the 'Engineering nonlinearity' method makes it possible to differentially reduce the stiffness parameters of sections of reinforced concrete elements [7, 8, 9].

In LIRA-SAPR 2018, two variants of the 'Engineering nonlinearity' method have been developed. In the first variant ('Engineering nonlinearity 1'), it is proposed to define reduced stiffnesses on the basis of the iterative method of analysis with the selection of reinforcement during this process. In the second variant ('Engineering nonlinearity 2'), reduced stiffness is determined by the step-type method with further defining of reinforcement. On the one hand, the 'Engineering nonlinearity 2' method allows you to take into account the physically nonlinear behaviour of the reinforced concrete section, on the other hand - to carry out analysis according to the standard procedure with account of DCF and DCL.

Analysis by 'Engineering Nonlinearity' method consists of two main stages. At the first stage, the static analysis of structure is carried out in characteristic load case (or combination of loads) that, by engineer's opinion, will have a significant effect on the stiffness of structure: crack development, plastic strain in concrete [7]. At the second stage, based on the analysis results, the stiffness values are modified according to the stress-strain state of each section and the standard analysis is carried out for the structure in which the elements have stiffness parameters determined at the first stage. Standard analysis includes linear-elastic analysis for the entire set of loads (dead weight, live load, earthquake, etc.), composing of DCF or DCL, analysis for sections of RC and steel elements, and design.

2. Stiffness parameters of bar section. Arbitrary section of bar is presented in Figure 1. Two moments M_x and M_y as well as axial force N act on the section. Moments act relative to principal axes of the section x and y. Axial force is applied at the point C – intersection between geometric axis of the bar and plane of the section. It is necessary to determine stiffness parameters of the section that correspond to secant moduli of elasticity of concrete and reinforcement.

To determine the stress-strain state, it is necessary to find out location of the neutral axis. This location is described with two values y_c, β and the curvature of the section ξ (see Fig.1): y_c – displacement of the neutral axis; β – rotation angle of the neutral axis; ξ – curvature of the section.

The problem is solved by numerical method. In iteration process three unknowns y_c, β, ξ, are determined; they are computed from three equations of equilibrium:

$$\sum z = 0\,,\ \sum M_x = 0\,,\ \sum M_y = 0\,,$$

$$\sum z = \sum_{j=1}^{n} \Delta F_{jб} \cdot \sigma_{jб}(y_c,\beta,\xi) + \sum_{i=1}^{m} f_{ia}\sigma_{ia}(y_c,\beta,\xi) + N = 0,$$

$$\sum M_x = \sum_{j=1}^{n} \Delta F_{jб} \cdot \sigma_{jб}(y_c,\beta,\xi)\, y_j(y_c,\beta,\xi) + \sum_{i=1}^{m} f_{ia}\sigma_{ia}(y_c,\beta,\xi)\, y_{ia}(y_c,\beta,\xi) + M_x + Ne_x = 0,$$

$$\sum M_y = \sum_{j=1}^{n} \Delta F_{jб} \cdot \sigma_{jб}(y_c, \beta, \xi) \cdot x_j(y_c, \beta, \xi) + \sum_{i=1}^{m} f_{ia} \sigma_{ia}(y_c, \beta, \xi) \cdot x_{ia}(y_c, \beta, \xi) + M_y + Ne_y = 0.$$

Stiffness parameters $E_{об}F$, $E_{об}I_x$, $E_{об}I_y$ are determined according to diagrams σ–ε for concrete and reinforcement. For concrete, only compressed part of concrete is considered with secant modulus of elasticity that is variable along the section. For every rebar, appropriate modulus of elasticity is used

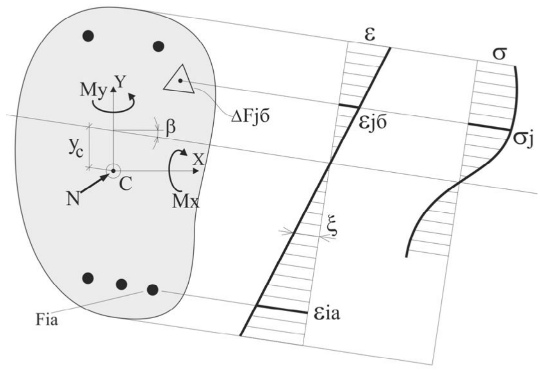

Fig. 1. Stress-strain state of the bar section

$$E_{об}F = \sum_{j=1}^{n} E_{секjб} \Delta F_{jб} + \sum_{i=1}^{m} E_{секia} f_{ia},$$

$$E_{об}I_x = \sum_{j=1}^{n} E_{секjб} \Delta F_{jб} y_{jб}^2 + \sum_{i=1}^{m} E_{секia} f_{ia} y_{ia}^2,$$

$$E_{об}I_y = \sum_{j=1}^{n} E_{секjб} \Delta F_{jб} x_{jб}^2 + \sum_{i=1}^{m} E_{секia} f_{ia} x_{ia}^2.$$

Here $\Delta F_{jб}$, f_{ia} – elementary zones to which section of concrete and areas of separate rebars are divided, n – number of zones in concrete, m – number of rebars, $E_{секjб}$, $E_{секia}$ – secant moduli of elasticity for concrete and reinforcement, they are determined according to diagrams σ–ε (see Fig. 1), $x_{jб}$, $y_{jб}$, x_{ia}, y_{ia} – distance from the gravity centre of the j-th zone of concrete and the i-th zone of rebar up to principal axes; location of principal axes (y_c, β) is determined during iteration analysis.

For concrete, only compressed part of concrete is considered with secant modulus of elasticity that is variable along the section. For every rebar, appropriate modulus of elasticity is used.

Stiffness matrix for the bar that has secant stiffness parameters variable along the length is also generated by numerical method (every bar is considered as a super-element).

This approach makes it possible to use the ‘Engineering nonlinearity 2’ method for analysis and modelling the erection process, analysis of panel buildings (platform joints), etc. In modelling the erection process, it is possible to specify characteristic load cases at stages. In a certain sense, ‘engineering nonlinearity 2’ transfers ideas of modelling the loading history (step-type method) to the analysis according to the standard type with account of physical nonlinearity.

Table 1 compares the features of the method for both variants.

Table 1

Comparison study for peculiar features of ‘Engineering nonlinearity’ method

Parameters	Engineering nonlinearity	Engineering nonlinearity 2
Characteristic load case	Arbitrary load cases are included according to the views of users as to their characteristic influence on the change (reduction) in stiffness of the element	Only long-term loads are included, further they will be included in the DCF and DCL
Methods for determining of reduced stiffness	Iteration	Step-type
Reinforcement	Determined during iteration analysis on characteristic load	Defined
Computing forces (analysis by standard type)	Analysis is carried out for all load cases according to secant moduli with further composing DCF, DCL	Analysis is carried out only on temporary load cases according to tangent moduli that correspond to the last step of the step-type analysis on characteristic load case. Results of this analysis as well as analysis results on characteristic load case are included into DCF and DCL
Account of physical nonlinearity during erection	Not available	Available
Account of nonlinear behaviour of platform joints (large panel buildings)	Not available	Available

3. Test example

Below there are analysis results for the test problem on the basis of engineering nonlinearity (Fig. 2). The load is q = 8 *t/rm* is accepted as the ‘characteristic load case’, temporary load is assumed to be 2 *t/rm*. Table 2 shows the analysis results for the frame with account of differentiated distribution of stiffnesses by different methods.

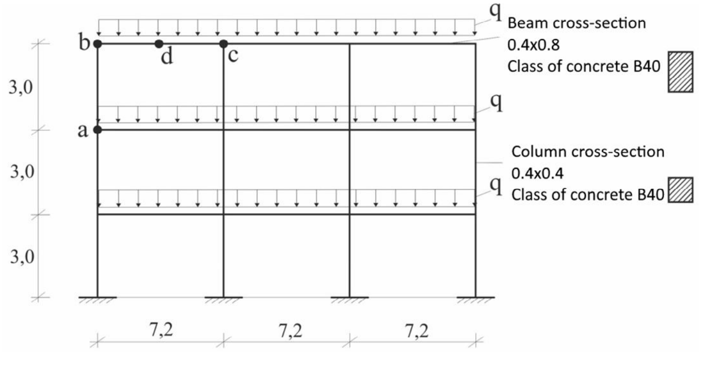

Fig. 2. Design model of the structure

Table 2

Analysis results of test example

Values for parameters of stress-strain state Type of analysis	Static analysis			Dynamic analysis	
	Moment in girder 'b–c' at node 'b', *tm*	Moment in girder 'b– ' at node 'd', *tm*	Displacement at node 'd', *tm*	Frequency ω, *Hz*	Period *T, sec*
Linear-elastic analysis with initial stiffnesses	-15.5	34	-3.95	1.56	0.64
Linear-elastic analysis with stiffnesses that are recommended in SP 52-103-2007	-20.9	30.8	-9.39	1.09	0.91
Linear-elastic analysis with stiffnesses obtained in the mode 'Engineering Nonlinearity 1'	-15,7	33,7	-5,02	1.65	0,61
Nonlinear analysis with stiffnesses in the mode 'Engineering Nonlinearity 2'	-16,5	32,9	-5,99	1,32	0,76
Physical nonlinearity	-16,5	32,9	-6,07		

Conclusions: By analysis results, we can draw the following conclusions:

- there is no significant redistribution of forces, it is explained by a small working load, though there is a tendency to redistribution - the smaller moment at node b increases, and the larger one at node d decreases;

- account of physical nonlinearity on displacements is more significant. It is possible to take the analysis by the step-type method (the 'physical nonlinearity' line) as the standard here.

Displacements at node *d* are increased 1.5 times in comparison with linear analysis. The 'Engineering nonlinearity' method shows approximately the same result (difference of 1.3%).

REFERENCES

1. *Bazhenov V.A. Gulyar O.I., Kozak O.L., Rutkovskyy V.A. Sakharov O.S.* Numerical modeling of concrete structures destruction by the finite element method // Kiev: Naukova dumka, 1996. – 360 p.
2. *Saharov A.S., Kislookij V.N., Kirichevskij V.V.* The finite elements method into mechanics of rigid bodies [The finite elements method into mechanics of rigid bodies]. Kiev: Vishha shk., 1982. – 480 p.
3. *Bazhenov V.A., Semenuk M.P., Trach V.M.* Neliniyne deformuvannya, stiykist' i zakritichna povedinka anizotropnih obolonok [Nonlinear deformation, stability and overcritical behavior of anisotropic shells]: Monographya. – K.: Karavella.- 2010. – 352 p.
4. *Bondarenko V.M., Bondarenko S.V.* Inzhenernye metody nelinejnoj teorii zhelezobetona [Engineering methods of nonlinear theory of reinforced concrete]. Moscow: Stroyizdat publ., 1982 – 287 p.
5. *Gorodetsky A.S.* Raschet konstrukcij na JeCVM s uchetom uprugo– plasticheskih deformacij [Analysis of structures on electronic digital computers (ETSVM) with account of elasto-plastic strain] // Proceedings from Report at the III National conference on application of ESTVM in building mechanics. Leningrad: Lenstroyizdat,1967.
6. *Barabash M.S.* Komp'juternoe modelirovanie processov zhiznennogo cikla ob'ektov stroitel'stva [Computer modelling of life cycle process for the building objects]. Kiev: Stal' publ.,2014 – 301 p.

7. *Barabash M.S.* Chislennoe modelirovanie NDS konstrukcij s uchetom stadij zhiznennogo cikla zdanij i sooruzhenij [Numerical modelling of the stress-strain state of structures with account of life cycle stages for buildings] // International Journal for Computational Civil and Structural Engineering, 2015.– v ol. 11, Issue 1. – P.80-90
8. *Gorodetsky A.S., Barabash M.S., Sidorov V.N.* Komp'juternoe modelirovanie processa vozvedenija stroitel'nyh konstrukcij [Computer modelling in problems of building mechanics]. Moscow: ASV publ., 2016 - 338 p.
9. *Gorodecky A.S., Barabash M.S.* Uchet nelinejnoj raboty zhelezobetona v PK LIRA-SAPR. Metod «Inzhenernaja nelinejnost'» [Account of nonlinear behaviour of reinforced concrete in LIRA-SAPR. Method 'Engineering Nonlinearity'] // International Journal for Computational Civil and Structural Engineering. – 2016.–Vol. 12, Issue 2. –P. 92-98

Стаття надійшла 24.05.2018

УДК 539.3:004.94: 624.074
Барабаш М.С.

ДЕЯКІ АСПЕКТИ МОДЕЛЮВАННЯ НЕЛІНІЙНОЇ РОБОТИ ЗАЛІЗОБЕТОНУ

У статті розглядаються деякі аспекти моделювання нелінійної роботи залізобетонних конструкцій, а саме метод «Інженерна нелінійність». Метод «Інженерна нелінійність» дозволяє розглядати фізично нелінійну поведінку залізобетонного перерізу ітераційним і кроковим методом. Метод дозволяє визначити параметри зниженою жорсткості перерізу, яка виникає через утворення тріщин, пластичні деформації в бетоні та арматурі. Описана концепція методу. Запропонований метод «Інженерна нелінійність» дозволяє інженеру більш точно оцінювати розподіл жорсткості. Цей метод майже аналогічний стандартним методам лінійного аналізу, а саме дозволяє проводити аналіз за всіма типами навантажень, складати розрахункові сполучення зксиль та розрахункові сполучення навантажень, аналізувати зусилля. Представлені результати аналізу тестової задачі, які показують певний перерозподіл сил і збіжність результатів, отриманих методом «Інженерна нелінійність» і з урахуванням фізичної нелінійності. Метод «Інженерна нелінійність» можливо використовувати для обчислення і моделювання процесу зведення, аналізу платформних стиків панельних будинків і таке інше.

Ключові слова: фізична нелінійність, інженерна нелінійність, життєвий цикл, модель проектування, комп'ютерне моделювання.

УДК 539.3:004.94: 624.074
Барабаш М.С.

НЕКОТОРЫЕ АСПЕКТЫ МОДЕЛИРОВАНИЯ НЕЛИНЕЙНОЙ РАБОТЫ ЖЕЛЕЗОБЕТОНА

В статье рассматриваются некоторые аспекты моделирования нелинейной работы железобетонных конструкций, а именно метод «Инженерная нелинейность». Метод «Инженерная нелинейность» позволяет рассматривать физически нелинейное поведение железобетонного сечения итерационным и шаговым методом. Метод позволяет определить параметры пониженной жесткости сечения, которая возникает из-за трещинообразования, пластической деформации в бетоне и арматуре. Описана концепция метода. Предлагаемый метод «инженерная нелинейность» позволяет инженеру более точно оценивать распределение жесткости. Этот метод почти аналогичен стандартным методам линейного анализа, т. е. позволяет проводить анализ по всем типам нагрузок, составлять РСУ и РСН, анализировать усилия. Представлены результаты анализа тестовой задачи (основанной на инженерной нелинейности), которые показывают некоторое перераспределение сил и сходимость результатов, полученных методом «Инженерная нелинейность» и с учетом физической нелинейности. Метод «Инженерная нелинейность» возможно использовать для вычисления и моделирования процесса возведения, анализа платформенных стыков панельных зданий и т.д.

Ключевые слова: физическая нелинейность, инженерная нелинейность, жизненный цикл, модель проектирования, компьютерное моделирование.

УДК 539.3:004.94: 624.074
Барабаш М.С. **Деякі аспекти моделювання нелінійної роботи залізобетону** // Опір матеріалів і теорія споруд: наук.-техн. збірник. – К.: КНУБА, 2018. – Вип. 100. – С. 164-171.

Представлено нові дослідження, пов'язані з виконанням розрахунків з урахуванням реальних властивостей матеріалу. Більшою мірою це стосується залізобетону, який вже при експлуатаційних навантаженнях в зв'язку з розвитком тріщин і пластичних деформацій бетону обумовлює значне зниження жорсткостей елементів і збільшення переміщень у порівнянні з розрахунком в лінійній постановці.
Табл. 2. Іл. 2. Бібліогр. 9 назв.

Barabash M. **Some aspects of modelling nonlinear behaviour of reinforced concrete** // Strength of Materials and Theory of Structures: Scientific-and-technical collected articles – Kyiv: KNUBA, 2018. – Issue 100. – P. 164-171.

New researches connected with execution of calculations taking into account real properties of a material are presented. To a greater extent, this relates to reinforced concrete, which already at operational loads due to the development of cracks and plastic deformations of concrete causes a significant reduction in the stiffness of the elements and increase displacement compared with the calculation in linear formulation.
Tables 2. Fig. 2. Ref. 9.

Барабаш М.С. **Некоторые аспекты моделирования нелинейной работы железобетона** // Сопротивление материалов и теория сооружений: науч.-техн. сборник. – К.: КНУСА, 2018. - Вып. 100. - С. 164-171.

Представлены новые исследования, связанные с выполнением расчетов с учетом реальных свойств материала. В большей степени это касается железобетона, который уже при эксплуатационных нагрузках в связи с развитием трещин и пластических деформаций бетона обусловливает значительное снижение жесткостей элементов и увеличение перемещений по сравнению с расчетом в линейной постановке.
Табл. 2. Ил. 2. Библиогр. 8 назв.

Автор (науковий ступінь, вчене звання, посада): *доктор технічних наук, доцент, старший науковий співробітник, професор кафедри комп'ютерних технологій будівництва Навчально-наукового інституту аеропортів НАУ БАРАБАШ Марія Сергіївна.*
Адреса робоча: *03058, Україна, м. Київ, просп. Космонавта Комарова 1, Національний авіаційний університет, Навчально-науковий інститут аеропортів. БАРАБАШ Марії Сергіївні*
Робочий тел.: *+38(044) 406-74-24.*
Мобільний тел.: *+38(095) 286-39-90.*
E-mail: bmari@ukr.net
ORCID ID: https://orcid.org/0000-0003-2157-521X