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A METHOD OF DETERMINING THE COORDINATES OF THE STIFFNESS CENTER AND THE STIFFNESS PRINCIPAL AXIS OF THE VIBRATING SYSTEM WITH DAMPING

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The report presents a methodology to determine the directions of the stiffness principal axis (in this case subject to the linear displacement and forced rotation angle) of a solid object interact with the surrounding environment by resilient bearing supports.

The results also show that determining the coordinates of the stiffness center in the vibrating system with damping factors is necessary in our research.

Key words: stiffness center, stiffness principal axis.

1. Introduction

Consider a three dimensional deformable body without vibration (Figure 1). The motion of an object so in the general case of six degrees of freedom are described by six independent geometric parameters that we can choose not only a unique way.

Suppose not vibrating objects has a mass of *m* and *n* is the number of springs (resilient bearing supports) attached to the immovable hard foundtion. Assume that *x*, *y*, *z* coordinate system to the real space origin at point *O*. At the same time we also assume that there exists a coordinate system tied to mobile *oxyz* shaking objects in question and at the initial $t=0$ coincide with the real coordinates *OXYZ*.

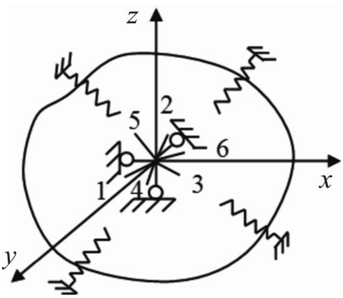

Fig. 1. A three dimensional deformable body without vibration

Suppose the object is doing a survey translational motion and at that time, the coordinate system *oxyz* is moving to a new location *o'x'y'z'*. The origin *O* moved to the *O'* and at that time, suppose the origin *O* will make a move with the components in the *X*,*Y*,*Z* respectively by ξ_0, η_0, ζ_0. Also solid in the new location have made the transposition angle about the axis X_1, Y_1, Z_3 respectively parallel to the axes X,Y,Z and passing through point O', the corresponding angle in φ_x, φ_y and φ_z.

In summary, six independent parameters ξ_0, η_0, ζ_0, φ_x, φ_y, φ_z completely determining the position of objects in space are examined at each time. If the object affected by the force $\overline{N}$ and moment $\overline{M}$, they will cause the displacement directly by the axes X,Y,Z and move the angle around the axis passing through O' and parallel to the X,Y,Z. If the object only affected by the force $\overline{N}$ with arbitrary set points, or only under the action of moment $\overline{M}$, one of these two factors will cause objects to both the linear displacement and rotation angle.

We will prove that in some cases the object exists on a point which, if put into force $\overline{N}$ and moment $\overline{M}$, then force only caused by the force linear displacement, moment only cause rotation angle around this point. That point is called the stiffness center in the vibrating system with damping. In other words, stiffness center of a vibrating system with damping factor considered is the set of all forces of the reaction components as objects to explore a dynamic loads by any way. At that time, the assumption will survive the stiffness axis of the solid for both linear displacement and the rotation angle, which this stiffness axis through the center of the vibrating system. The stiffness principal axis is characterized, if the force $\overline{N}$ acting on way solid survey coincides with way stiffness principal axis, it will only cause linear displacement under way this axis. Similarly the stiffness principal axis also exist for the rotation angle, meaning that if solid only under the action of an axis moment $\overline{M}$ is identical to the stiffness principal axis, it will only cause the angle around this axis.

2. Formulation of the problem

We will prove the above properties by means of the method of displacements. Assuming that orthogonal axes X,Y,Z have point located at the O. In this coordinate system the solid has six degrees of freedom. We put in six joints directly prevent displacement and rotation angle, assuming the first three joints 1,2,3 prevent the displacement and three linear following joints 4,5,6 prevent rotation angle (around the axes X,Y,Z). If the solid the unit forced displacement $\Delta x=1$, $\Delta y=1$ and $\Delta z=1$ by the way of the axes X,Y,Z respectively, then at the joints above, the jet will appear:

$$r_{11}, r_{21}, r_{31}, r_{41}, r_{51}, r_{61} \text{ respectively, by } \Delta_x = 1 \text{ caused;}$$
$$r_{12}, r_{22}, r_{32}, r_{42}, r_{52}, r_{62} \text{ respectively, by } \Delta_y = 1 \text{ caused;}$$
$$r_{13}, r_{23}, r_{33}, r_{43}, r_{53}, r_{63} \text{ respectively, by } \Delta_z = 1 \text{ caused.} \qquad (1)$$

Subsequently, for the solid angle surveyed rotation force units around the axis X,Y,Z, then the joint will appear jet components corresponding:

$$r_{14}, r_{24}, r_{34}, r_{44}, r_{54}, r_{64} \text{ respectively, by } \varphi_x = 1 \text{ caused;}$$

$r_{15}, r_{25}, r_{35}, r_{45}, r_{55}, r_{65}$ respectively, by $\varphi_y = 1$ caused;
$r_{16}, r_{26}, r_{36}, r_{46}, r_{56}, r_{66}$ respectively, by $\varphi_z = 1$ caused. (2)

Here, r_{ij} is the corresponding reaction appearing at the joint i by forced displacement unit according to the joint j caused.

We see $r_{ij} = r_{ji}$ the reciprocal theorem of the reaction, i = 1,2,3,4,5,6 and j = 1,2,3,4,5,6.

Assuming that we put into four matrix:

$$\overline{A}=\begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix};\ \overline{B}=\begin{bmatrix} r_{14} & r_{15} & r_{16} \\ r_{24} & r_{25} & r_{26} \\ r_{34} & r_{35} & r_{36} \end{bmatrix};$$

$$\overline{C}=\begin{bmatrix} r_{41} & r_{42} & r_{43} \\ r_{51} & r_{52} & r_{53} \\ r_{61} & r_{62} & r_{63} \end{bmatrix};\ \overline{D}=\begin{bmatrix} r_{44} & r_{45} & r_{46} \\ r_{54} & r_{55} & r_{56} \\ r_{64} & r_{65} & r_{66} \end{bmatrix}. \tag{3}$$

From reciprocity theorem, we have $\overline{B} = \overline{C}$. Assuming that force $\overline{N}$ and moment $\overline{M}$, that they have the way and know beforehand the numbers, they cause the linear displacement ξ, η and ζ of a solid under the way X, Y, Z and the rotation angle φ_x, φ_y and φ_z around X,Y,Z respectively.

In this case, we have the relationship between external forces and displacements in the matrix

$$\left.\begin{aligned} \overline{N} &= \overline{A}\overline{W} + \overline{B}\overline{\varphi} \\ \overline{M} &= \overline{C}\overline{W} + \overline{D}\overline{\varphi} \end{aligned}\right\} \tag{4}$$

With $\overline{W}$ - the total linear displacement vector; $\overline{\varphi}$ - the total rotation angle vector.

To the way $\overline{W}$ and $\overline{N}$ of the same, should be conditional:

$$\overline{N} = \lambda\overline{W};\ \overline{M} = \upsilon\overline{\varphi}. \tag{5}$$

With λ and υ are the scalar magnitudes:

Replace the value of $\overline{N}$ and $\overline{M}$ (5) into (4) we have:

$$\left.\begin{aligned} (\overline{A} - \lambda\overline{I})\overline{W} + \overline{B}\overline{\varphi} &= 0 \\ \overline{C}\overline{W} + (\overline{D} - \upsilon\overline{I})\overline{\varphi} &= 0 \end{aligned}\right\}. \tag{6}$$

With $\overline{I}$ is the ranked third matrix units

$$\overline{I}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

We woul'd like interpretate above proposition as following: the elements of the matrix $\overline{A}$ independent original position that coordinates the concurrent joint there. It just depends on the the way of the coordinate axes coincide with the way of the obstacles joint of a solid linear displacement, that we are surveying.

In fact, elements of the matrix $\overline{A}$ shows reaction at the joints directly obstructing the displacement of a solid as a result of the unit forced displacement of solids by the way of joints. The components of this force can be found by solving the balance equation generation, each one of these equations would shows that the total elevation of all forces acting on the solid to the way axis with the way of parallel joints, where we need to determine the composition of reaction. Clearly, the value of the reaction depends only on the way joint that does not depend on the located joint. Therefore, the elements of the matrix $\overline{A}$ depends only on the way joint linear obstructing the displacement of a solid.

We easily see that the elements of the matrix $\overline{D}$ has no properties as the matrix $\overline{A}$. If we choose O as the starting point of the joint located, the elements of the matrix $\overline{B}$ is zero and according to reciprocity theorem, we find the matrix elements $\overline{C}$ also zero. At that time (6) takes the form:

$$(\overline{A}-\lambda\overline{I})\overline{W}=0\,;\;(\overline{D}-\upsilon\overline{I})\overline{\varphi}=0\,. \tag{7}$$

Equation (7) is to ensure conditions for the existence of O stiffness center vibrating system damping factor considered. This means that only exists a point O of the solid: If set to a force applying point $\overline{N}$ (passing O) and a moment $\overline{M}$ (with axis passing through O), the force caused only by the linear displacement of the force, moment is only caused rotation displacement around the point O. The point O is the stiffness center of the system vibration damping.

Formula (7) allows to determine the three values of λ and the three values of υ. Knowing λ and υ can be determined three directional cosins for λ and three directional cosins to υ. Therefore directional cosins of three directional mutual perpendicular to axis of coordinates. These ways are the directional of the principal axis to the linear displacement and the rotation angle of the solid. Indeed, from the first formula of (7) we can locate the principal axis of the solid linear displacement. We have:

$$\begin{bmatrix} r_{11}-\lambda & r_{12} & r_{13} \\ r_{21} & r_{22}-\lambda & r_{23} \\ r_{31} & r_{32} & r_{33}-\lambda \end{bmatrix} \begin{bmatrix} W\cos\alpha \\ W\cos\beta \\ W\cos\gamma \end{bmatrix}. \tag{8}$$

Here *W*cosα, *W*cosβ, *W*cosγ are the components of vector $\vec{W}$ of the linear displacement; α, β, γ are the angle of the vector by ways of total rotation with the axes *X*, *Y*, *Z* respectively.

We represent (8) in algebraic form:

$$\left.\begin{aligned}(r_{11}-\lambda)\cos\alpha+r_{12}\cos\beta+r_{13}\cos\gamma=0\\ r_{21}\cos\alpha+(r_{22}-\lambda)\cos\beta+r_{23}\cos\gamma=0\\ r_{31}\cos\alpha+r_{32}\cos\beta+(r_{33}-\lambda)\cos\gamma=0\end{aligned}\right\}. \quad (9)$$

Indeed, it is the homogeneous equation in the directional cosins. To obtain non-trivial solution of algebraic equations we have the following condition det(...)=0, i.e.condition on zero determinant of coefficients of unknowns of algebraic equations system.

This leads to the cubic equation:

$$\lambda^3+b_1\lambda^2+c_1\lambda+d_1=0 . \quad (10)$$

With: $b_1=-r_{11}-r_{22}-r_{33}$; $c_1=-r_{11}r_{33}+r_{22}r_{33}+r_{11}r_{22}-r_{12}^2-r_{23}^2-r_{13}^2$;

$d_1=-r_{11}r_{23}^2+r_{22}r_{13}^2+r_{33}r_{12}^2-r_{11}r_{22}r_{33}-2r_{12}r_{31}r_{23}$.

Solving the equation (10) may be obtained:

$$\lambda_1=u_1+v_1-\frac{b_1}{3} ;\ \lambda_2=\varepsilon_1u_1+\varepsilon_2v_1 ;\ \lambda_3=\varepsilon_2u_1+\varepsilon_1v_1 . \quad (11)$$

With: $u_1=\sqrt[3]{-q_1+\sqrt{q_1^2+p_1^3}}$; $v_1=\sqrt[3]{-q_1^2-\sqrt{q_1^2+p_1^3}}$; $\xi_{1,2}=-\frac{1}{2}\pm i\frac{\sqrt{3}}{2}$; $i=\sqrt{-1}$;

$q_1=\frac{b_1^3}{27}-\frac{b_1c_1}{6}+\frac{d_1}{2}$; $v_1=\sqrt[3]{-q_1^2-\sqrt{q_1^2+p_1^3}}$; $p_1=\frac{3c_1-b_1^2}{9}$.

In addition,

$$\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1 . \quad (12)$$

Solve (9) in the hiddens $\frac{\cos\beta}{\cos\alpha}$ and $\frac{\cos\gamma}{\cos\alpha}$ for earch λ_j, we have:

$$\frac{\cos\beta_j}{\cos\alpha_j}=p_j ;\ \frac{\cos\gamma_j}{\cos\alpha_j}=\xi_j . \quad (13)$$

With: $p_j=\frac{r_{12}r_{13}-r_{23}(r_{11}-\lambda_j)}{r_{12}r_{23}-r_{13}(r_{22}-\lambda_j)}=\frac{r_{13}r_{23}-r_{12}(r_{33}-\lambda_j)}{(r_{22}-\lambda_j)(r_{33}-\lambda_j)-r_{23}^2}=\frac{r_{13}^2-(r_{11}-\lambda_j)(r_{33}-\lambda_j)}{r_{12}(r_{33}-\lambda_j)-r_{13}r_{23}}$;

$\xi_j=\frac{r_{12}r_{13}-r_{23}(r_{11}-\lambda_j)}{r_{12}r_{23}-r_{12}(r_{33}-\lambda_j)}=\frac{r_{12}^2-(r_{11}-\lambda_j)(r_{22}-\lambda_j)}{r_{13}(r_{22}-\lambda_j)-r_{12}r_{23}}=\frac{r_{13}(r_{22}-\lambda_j)-r_{12}r_{32}}{r_{23}^2-(r_{22}-\lambda_j)(r_{33}-\lambda_j)}$;

$j=1,2,3$.

In summary to determine the cosin ways, we have a system of equations:

$$\cos\beta_i = p_i \cos\alpha_i \; ; \; \cos\gamma_j = \xi_j \cos\alpha_j \; ; \; \cos^2_{\alpha_j} + \cos^2\gamma_j + \cos^2_{\gamma_j} = 1 \, . \quad (14)$$

Solution (14) we have: $\cos\alpha_j = \pm\dfrac{1}{\sqrt{1+p_j^2+\xi_j^2}}$; $\cos\beta_j = \pm\dfrac{a_j}{\sqrt{1+p_j^2+\xi_j^2}}$;

$\cos\gamma_j = \pm\dfrac{b_j}{\sqrt{1+\eta_j^2+\zeta_j^2}}$ to identify three stiffness principal axis to the linear displacement.

Similarly, we can determine directional cosins of the three stiffness principal axis, for the component rotation angle.

We have:

$$\cos\mu_j = \pm\frac{1}{\sqrt{1+\eta_j^2+\zeta_j^2}} \; ; \; \cos\theta_j = \pm\frac{\eta_j}{\sqrt{1+\eta_j^2+\zeta_j^2}} \; ; \; \cos\psi_j = \pm\frac{\zeta_i}{\sqrt{1+\eta_j^2+\zeta_j^2}} \, .$$

Here:

$$\eta_j = \frac{r_{46}r_{54} - (r_{44} - \upsilon_j)r_{56}}{r_{45}r_{56} - r_{46}(r_{55} - \upsilon_j)} = \frac{r_{46}r_{56} - (r_{66} - \upsilon_j)r_{54}}{(r_{55} - \upsilon_j)(r_{66} - \upsilon_j) - r_{56}^2} = \frac{r_{46}^2 - (r_{44} - \upsilon_j)(r_{56} - \upsilon_j)}{(r_{66} - \upsilon_j)r_{45} - r_{45}r_{65}} \; ;$$

$$\zeta_j = \frac{r_{45}^2 - (r_{44} - \upsilon_j)(r_{45} - \upsilon_j)}{(r_{55} - \upsilon_j)r_{46} - r_{45}r_{56}} = \frac{r_{64}(r_{55} - \upsilon_j)r_{54}r_{65}}{r_{56}^2 - (r_{55} - \upsilon_j)(r_{66} - \upsilon_j)} = \frac{r_{45}r_{64} - r_{65}(r_{44} - \upsilon_j)}{r_{46}r_{65} - r_{45}(r_{66} - \upsilon_j)}$$

With:

$$\upsilon_1 = u_2 + v_2 - \frac{b_2}{3} \; ; \; \upsilon_2 = \varepsilon_1 u_2 + \varepsilon_2 v_2 \; ; \; \upsilon_3 = \varepsilon_2 u_2 + \varepsilon_1 v_2 \; ; \; u_2 = \sqrt[3]{-q_2 + \sqrt{q_1^2 - p_2^2}} \; ;$$

$$v_2 = \sqrt[3]{-q_2 - \sqrt{q_2^2 - p_2^2}} \; ; \; q_2 = \frac{b_2^3}{27} - \frac{b_2 c_2}{6} + \frac{d_2}{2} \; ; \; p_2 = \frac{3c_2 - b_2^2}{9} \; ; \; b_2 = -r_{44} - r_{55} - r_{66} \; ;$$

$$c_2 = -r_{44}r_{66} + r_{55}r_{66} + r_{44}r_{55} - r_{45}^2 - r_{56}^2 - r_{46}^2 \; ;$$

$$d_2 = r_{44}r_{56}^2 + r_{55}r_{46}^2 + r_{66}r_{45}^2 - r_{44}r_{55}r_{66} - 2r_{45}r_{64}r_{56} \, . \quad (16)$$

Suppose X_k, Y_k, Z_k are the stiffness principal axis of the linear displacement (in O_k origin) are stiffness center of vibration damping system (Figure 2). Choose the rectangular coordinate system with the coordinate axes *OXYZ* parallel to the stiffness principal axis X_k, Y_k, Z_k (Figure 2). We dropped into the solid at the point O_k (stiffness center) six joints in the joints that hinder 1,2,3 of the solid linear displacement by ways of the stiffness principal X_k, Y_k, Z_k, three relevant the remaining 4,5,6 hinder the rotation angle of the solid around these axes.

For the solid displacement forced by ways units of the coordinate axes *X,Y,Z.* Then the forced displacement unit according to the X_k, Y_k, Z_k. Notice that the the axes X_k, Y_k, Z_k are the stiffness principal axis of the linear displacement, so we can see $\overline{B}=0$ và $\overline{C}=0$.

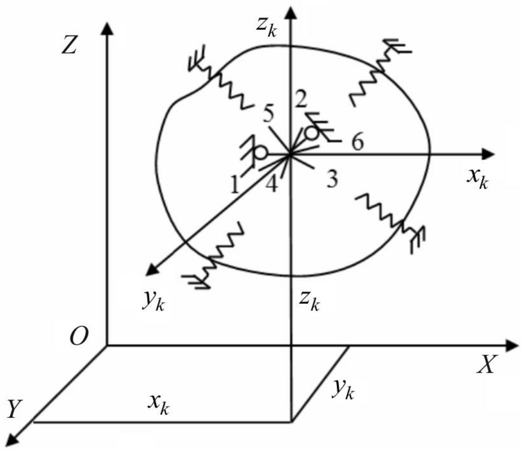

Fig. 2: Stiffness center of vibration damping system

We easily see that:

when $\Delta_x = 1$, all of $r_{ij} = 0$ except for $r_{11} = k_x$;
when $\Delta_y = 1$, all of $r_{ij} = 0$ except for $r_{22} = k_y$; (i = 1,2,3,4,5,6);
when $\Delta_z = 1$, all of $r_{ij} = 0$ except for $r_{33} = k_z$; (j = 1,2,3,4,5,6). (17)

While k_x, k_y, k_z are the general stiffness of the dampers (springs) relative to ways axes X, Y, Z.

Condition (17) allows us to determine the stiffness center coordinates of thevibrating system, when it is subjected to forced linear displacement and forced rotation angle. If we replace the shock absorber (the spring) by the elastic reaction. Then we make the total monent of the forces for the axis OY, while the solid under forced displacement unit according to the X ($\Delta x = 1$), then the sum are zero, we calculated:

$$z_k = \frac{\sum M_{0y}^*}{k_x} . \tag{18}$$

Similarly in the case of the solid under the forced displacement unit according to the Y ($\Delta y = 1$), we have:

$$z_k = \frac{\sum M_{0x}^*}{k_y} . \tag{19}$$

And if the solid under forced displacement unit according to the Z ($\Delta z = 1$), we have:

$$x_k = \frac{\sum M_{0y}}{k_z}; \; y_k = \frac{\sum M_{0x}}{k_z}. \quad (20)$$

In the formula (18), (19), (20) x_k, y_k, z_k are in the stiffness center coordinates of the coordinate system *OXYZ*. Which is the sum of the corresponding reaction moment which it appears in the spring dampers. It is taken for the axes *OX* and *OY* under the system of forced shift in the *X* and *Y*. That means that when $\Delta x = 1$ and $\Delta y = 1$. The total moment of the jet appears in the position of the spring damper taken for the axes *OX* and *OY* under the system of forced displacement unitaccording to the *z*, that means $\Delta z = 1$.

Suppose, the main (of the linear displacement and the rotation angle) which has identical origin at the stiffness center. When that force is ways same as one of the coordinate axes will cause linear displacement by ways of the cylinder. Simultaneously with the moment coincides with one of the coordinate system will only cause rotation angle around this axis.

3. Example

At the flat problem when vibrating system has three degrees of freedom, two linear displacement ways mutual, perpendicular and one rotate displacement in a plane. Assuming rectangular coordinate system origin at *O* of the solid (Figure 3).

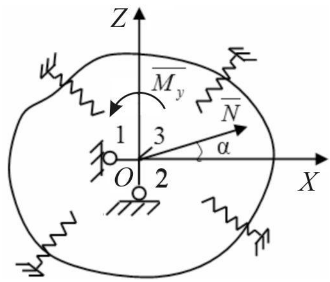

Fig. 3. Rectangular coordinate system origin at *O* of the solid

We put in the survey solid three joints located at point *O*, two of the joints will interfere with the linear displacement in the *X* and *Z*, while the remaining joints will impede the rotation displacement of the solid in *XOZ* plane.

Continue to the solid the linear and angle units displacements according to the ways of the joints included. At that time the joint will appear the jet, respectively by:

r_{11}, r_{21}, r_{31} when $\Delta_x = 1$; r_{12}, r_{22}, r_{32} when $\Delta_z = 1$; r_{13}, r_{23}, r_{33} when $\varphi_y = 1$. (21)

If we put force $\overline{N}$ and moment $\overline{M_y}$ on the solid, they will cause the transition (ξ, ζ) corresponding to ways *X*, *Z* and rotation angle φ in the plane *XOZ* around axis *Y*.

We have:

$$N_x = r_{11}\xi + r_{12}\eta + r_{13}\varphi; \; N_z = r_{21}\xi + r_{22}\eta + r_{23}\varphi; \; M_y = r_{31}\xi + r_{32}\eta + r_{33}\varphi. \quad (22)$$

At here: $Nx = N\cos\alpha$, $Nz = N\sin\alpha$ corresponding are the projection of force N onto the coordinate axes X and Z, and α is the angle by ways of the vector N to the axis OX.

If the coordinate system OXZ placed at the stiffness center of the vibrating system, then:

$$r_{13} = r_{23} = r_{31} = r_{32} = 0. \qquad (23)$$

If you look at (23), equations (22) has the form:

$$N_x = r_{11}\xi + r_{12}\eta\ ;\quad N_z = r_{21}\xi + r_{22}\eta\ ;\quad M_y = r_{33}\varphi \qquad (24)$$

To determine the stiffness principal axis, when the system under linear displacement, we have the conditions:

$$\xi = W\cos\alpha\ ;\quad \eta = W\sin\alpha\ ;\quad N_x = \lambda W\cos\alpha\ ;\quad N_z = \lambda W\sin\alpha \qquad (25)$$

With W – length total displacement vector, and λ are unknown quantity.

The conditions (25) gives us the related:

$$(r_{11} - \lambda)\cos\alpha + r_{12}\sin\alpha = 0\ ;\quad r_{21}\cos\alpha + r_{12}(r_{22} - \lambda)\sin\alpha = 0\ . \qquad (26)$$

Two equations (26) linearly independent if the determinant of the coefficients ($\sin\alpha$ and $\cos\alpha$) have zero value, ie:

$$(r_{11} - \lambda)(r_{22} - \lambda) - r_{21}r_{12} = 0\ . \qquad (27)$$

Inferred:

$$\lambda_{12} = \frac{r_{11} + r_{22}}{2} \pm \sqrt{\frac{(r_{11} - r_{22})^2}{4} + r_{12}^2}\ . \qquad (28)$$

Depending on the values of $\lambda_{1,\,2}$, we calculate the values of the angle α by the formula:

$$tg\alpha_{1,2} = -\frac{r_{11} - \lambda_{1,2}}{r_{12}} = -\frac{r_{21}}{r_{22} - \lambda_{1,2}}\ . \qquad (29)$$

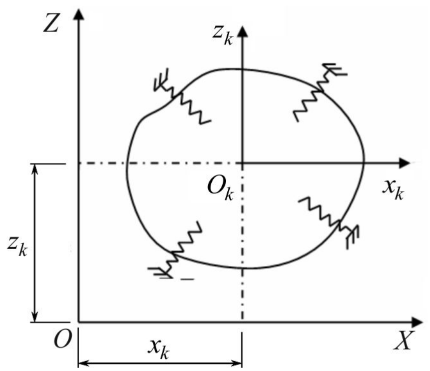

Fig. 4. The linear displacement, with original stiffness center in mind O_k

We find $tg\alpha_1 = -ctg\alpha_2$. This means the two stiffness principal axis thas found the mutual perpendicular.

Suppose X_k and Z_k respectively two stiffness principal axis of the linear displacement, with original stiffness center in mind O_k (Figure 4).

At that time (Figure 4) in the coordinate system OXZ, stiffness center coordinates can be found by the formula:

$$z_k = \frac{\sum M_{0y}^*}{k_x}\ ;\ x_k = \frac{\sum M_{0y}^0}{k_z}$$

With k_x and k_z are the stiffness of the spring damper according to the axes X and Z; $\sum M_{0y}^*$ - The sum of the moment reaction occurs in the spring damping, taking to the OY axis is perpendicular to the solid surface XOZ under displacement unit of the x axis, ie when $\Delta x = 1$; $\sum M_{0y}^0$ - The sum of the moment reaction appears in the position of the spring damping taken for the axis OY, but when the solid under forced transfer units according to the OZ axis, ie when $\Delta z = 1$.

4. Conclusion

We have observed that, the linear displacement and rotation angle of the solid markersare taken from a static equilibrium position, while only bear the weight of solid material itself. So the problem posed and studied here has a certain significance for marine gravity projects. If the displacement of a solid is small, the balance of the vibrating system is stable. Use wing vibration theory [1,2], the concept of the stiffness axis for linear displacement and rotation angle as well as the concept of center stiffness to imagine clearer picture of the oscillations on coastal structures fixed (rig, lighthouse tower ...) when subjected to the forced transfer by waves.

The problem is that the damping springs arranged logically necessary, so that their axes through the center of the stiffness system and the fluctuations coincide with the stiffness principal axis, the ability to preserve the stability vibration are very good and calculate the fluctuations are also a lot simpler.

In many cases it can replace the dynamic force and moment by the equivalent static force set points coincide with the center of the vibrating system stiffness. This makes sense from the point transfer only to the dynamic load bearing structure of the rig.

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МЕТОД ВИЗНАЧЕННЯ КООРДИНАТ ЦЕНТРУ ЖОРСТКОСТІ І ГОЛОВНОЇ ОСІ ЖОРСТКОСТІ КОЛИВАЛЬНОЇ СИСТЕМИ ІЗ ЗАГАСАННЯМ

У статті представлена методика визначення напрямів головної осі жорсткості у разі лінійного зсуву і примусового повороту твердого тіла, що взаємодіє з навколишнім середовищем через пружні опори.

Результати показують, що в нашому дослідженні коливальної системи з демпфуванням визначення координат центру жорсткості є необхідним.

Ключові слова: центр жорсткості, головна вісь жорсткості.

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МЕТОД ОПРЕДЕЛЕНИЯ КООРДИНАТ ЦЕНТРА ЖЕСТКОСТИ И ГЛАВНОЙ ОСИ ЖЕСТКОСТИ КОЛЕБАТЕЛЬНОЙ СИСТЕМЫ С ЗАТУХАНИЕМ

В статье представлена методика определения направлений главной оси жесткости в случае линейного смещения и принудительного поворота твердого тела, взаимодействующего с окружающей средой через упругие опоры.

Результаты показывают, что в нашем исследовании колебательной системы с демпфированием определение координат центра жесткости является необходимым.

Ключевые слова: центр жесткости, главная ось жесткости.