

UDC 539.3

DYNAMIC BEHAVIOUR OF NONLINEAR NONSMOOTH DISCONTINUOUS VIBROIMPACT SYSTEM

V.A. Bazhenov¹

Doctor of Technical Sciences, Professor, Academician of the Ukraine National Academy of Pedagogical Sciences

O.S. Pogorelova¹

Candidate of Physico-mathematical Sciences, Senior Research Officer, Senior Research Officer

T.G. Postnikova¹

Candidate of Engineering Sciences, Senior Research Officer, Senior Research Officer

¹*Kyiv National University of Construction and Architecture*

Authors shortly describe the problem state about studying of discontinuous vibroimpact systems dynamical behaviour. Recently the investigations of such systems are developed rapidly. There is survey of world scientific literature about this problem. Information and description of an International Conference on Nonlinear Dynamics Complexity is given. Authors show owns the most spectacular results demonstrating the phenomena unique for nonsmooth systems describing by differential equations with discontinuous right-hand side. These results were obtained by numerical parameter continuation method.

Keywords: vibro-impact system, discontinuous, conference, parametric continuation technique, instability, monodromy matrix, multipliers, bifurcation points

1. Introduction

Vibroimpact machines and equipment are often encountered in many engineering practice applications. Therefore their dynamic behavior is studied very intensively now. Big attention is paid to investigate stability in systems with impacts, periodic motions, bifurcations, singularities at vibroimpact dynamics [1-10].

Vibroimpact system is strongly nonlinear one and is the system with discontinuous right-hand side. Many dynamical systems arising in applications are non-smooth; examples include the occurrence of impacting motion in mechanical systems. Many new phenomena unique to non-smooth systems are observed under variation of system parameters. Jumps and switches in a system's state represent the grossest form of nonlinearity [11].

Recently the investigations of such systems are developed rapidly. At 2002 Professor Liene R.I. [12] wrote: "During the last decades many textbooks about bifurcation theory for smooth systems appeared and bifurcations of smooth vector fields are well understood [13]. However, little is known about bifurcations of discontinuous vector fields." At 2012 Professor Ivanov A.P. [14] already said that there were considerable publication numbers about bifurcations in non-smooth systems. He referred to survey [15] and monograph [11] by Prof. di Bernardo M..

In general there were the numerical investigations which constructed phase portraits for different parameters values.

Professors Liene [16] and di Bernardo M. [17] divide discontinuous dynamical systems into three types according to their degree of discontinuity. There are among them Filippov systems and the impacting systems with velocity reversals.

In review [15] three classes of systems are considered, involving either state jumps, jumps in the vector field or jumps in some derivative of the vector field. In this work there is the explanation of dynamics that may be observed in simulations of systems which include friction oscillators, impact oscillators, DC-DC converters, and problem in control theory. Authors note that in recent years there has been significant progress in identifying, classifying, unfolding and applying novel kinds of bifurcations that are unique to nonsmooth systems.

In [11] authors say that it has become increasingly clear that there are distinctive phenomena unique to discontinuous systems, which can be analyzed mathematically but fall outside the usual methodology for smooth dynamical systems. This book introduces a similar qualitative theory for non-smooth systems. Authors propose general techniques for analyzing the bifurcation that are unique to non-smooth dynamical systems, so-called discontinuity-induced bifurcations (DIBs). Authors give a consistent classification of all known DIBs for piecewise-smooth continuous-time dynamical systems (flows), including such diverse phenomena as sliding, chattering, grazing and corner collision.

The paper [17] proposes a strategy for the classification of codimension-two grazing bifurcations of limit cycles in piecewise-smooth systems of ordinary differential equations.

In [12,16] discontinuous systems of the Fillipov type were studied. Authors explain how jumps in the fundamental solution matrix lead to jumps of the Floquet multipliers of periodic solutions. A Floquet multiplier of a discontinuous system can jump through the unit circle causing a discontinuous bifurcation when a parameter of the system is varied. Discontinuous bifurcations come into being through jumps of the fundamental solution matrix. Discontinuous bifurcation is a novel, nonclassical type of bifurcation.

In contemporary monograph [5] authors pay big attention to investigation of discontinuous dynamical systems which extensively exist in engineering. In mechanical engineering there are two common and important contacts in dynamical systems, that is, impact and friction. The theory of discontinuous dynamical systems is presented as a base from which to study vibro-impact dynamics in engineering. In this book, a systematic way is developed through a few simple vibro-oscillators in order to understand the physics of vibro-impact systems in engineering.

In comprehensive works [18,19] dynamics of impacting systems is reviewed in depth. These works are devoted to study of rigid multi-body mechanical systems subject to nonsmooth effects, such as impacts, Coulomb friction, constraints addition and deletion.

In [20] author studies the dynamics of piecewise-smooth (PWS) systems. Of his particular interest are hybrid PWS. This class of dynamical systems can exhibit a rich bifurcation scenario involving the occurrence of both classical bifurcations (saddle-node, period-doubling, etc.) and so-called discontinuity-induced bifurcations (DIBs). DIBs are unique to piece-smooth dynamical systems. He writes that in impacting systems the most notable type of DIB is the grazing bifurcation of a limit cycle, observed when, under parameter variations, a limit cycle becomes tangential to the system discontinuity manifold. In [21] have been shown that grazing bifurcations are associated to a wide range of dynamical transitions including nonsmooth folds and sudden transitions from periodic to chaotic behavior. The study of the system was performed by combining experimental, numerical and analytical procedures that allowed to unveil the coexistence between a classical period-doubling route to chaos and a novel discontinuity-induced bifurcation phenomenon generated at interruption of a periodic complete-chattering motion.

In [14] author considers dynamical systems with discontinuous right-hand side. A method of stepwise is proposed allowing the reduction of discontinuous bifurcations to a sequence of typical bifurcations: saddle-node, period-doubling and Hopf bifurcations.

At our works we apply the numerical parameter continuation method in order to study dynamic behaviour and periodic motions stability for specific discontinuous vibroimpact system, to find the bifurcation points. We use parametric continuation technique in conjunction with shooting and Newton-Raphson methods [22].

We have watched the phenomena unique for nonsmooth discontinuous system under considering specific two-body two-degree-of-freedom vibroimpact system with discontinuous right-hand side. We have observed discontinuous bifurcation points where set-valued Floquet multipliers cross the unit circle by jump. At these points monodromy matrix is changed by jump too. We also have observed chattering regimes leading to chaos [23,24].

These results were the contents of our contribution into an International Conference on Nonlinear Dynamics Complexity [25]. Below at section 3 we'll show the most spectacular results.

2. Information about an International Conference on Nonlinear Dynamics Complexity

In May 2015 during 5 days there was an International Conference on Nonlinear Dynamics Complexity in Spain. The Conference was small but very interesting. There were 40 reports. Many of them were made at high scientific level. The Conference was represented by scientists from many countries: Spain, Germany, Egypt, Mexico, Turkey, Poland, China, France, India, Russia, Algeria, USA, Romania, Ukraine, UK, Portugal, and Brazil. Ukraine was represented by our talk “Bifurcation Points under 2-DOF Vibroimpact System Moving. Numerical Analysis by Parameter Continuation Method” by V.A.Bazhenov, P.P.Lizunov, O.S.Pogorelova, T.G.Postnikova. (Fig. 1).

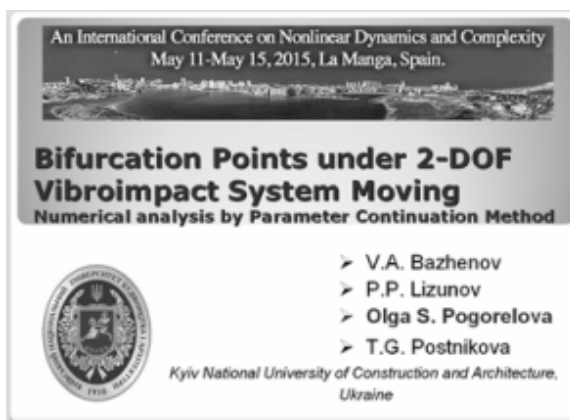


Fig. 1

There were two well talks from Russia. There were two scientists who had lived in USSR before but now they represented other countries (Mexico and Turkey).

The General chair of Organizing Committee was Full Professor of Applied Mathematics from Universidal Politecnica de Cartagena Juan Luis Garcia Guirao. The Co-Chair of Organizing Committee was Professor of Southern Illinois University (USA) Albert C.J. Luo. Note that Professor Albert C.J.Luo and Coordinator Professor of ISEP-Institute of Engineering of Porto (Portugal) J.A. Tenreiro Machado are Editors of “Journal of Applied Nonlinear Dynamics”. We want to note by the way that Professor Albert Luo did not omit single report during conference. There is photo of participants near Universidal Politecnica de Cartagena (Fig.2). Olga S.Pogorelova, Professor Juan Luis Garcia Guirao, Professor James Yorke (USA), and Professor Mahmoud Abdel-Aty (Egypt) are sitting on the bench. Professor Tenreiro Machado is sitting on

the floor. Professor Albert Luo in blue shirt is standing behind Juan L.G. Guirao.



Fig. 2

At short meeting in Universidal Politecnica de Cartagena President of University and Chief of Department of Applied Mathematics have talked the short speeches and have welcomed the Conference participants. Both of them spoke English freely (Fig 3). The Conference President Juan Luis Garcia Guirao is left.



Fig. 3

Our report was successfully talking by Olga S. Pogorelova (Fig.4). We placed an Ukrainian girl at our last poster. Professor James Yorke asked who this girl is.



Fig.4

There is Certifies about our talk delivering (Fig.5).

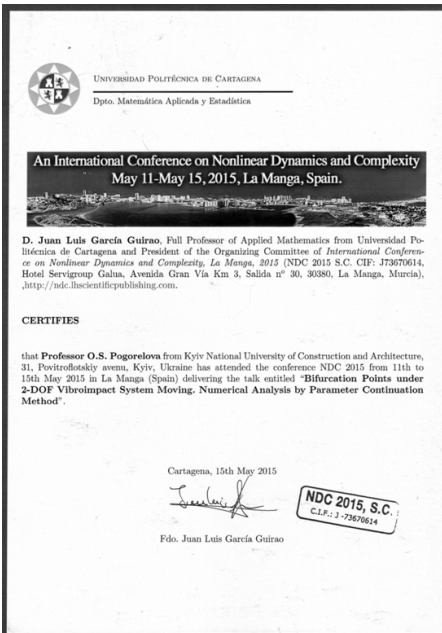


Fig. 5

Professor from Egypt Mahmoud Abdel-Aty delivered the first plenary talk “Multi-Qubit Dynamics Quantum Search Algorithm”. He proposed a new physical scheme for perfect implementation of quantum search algorithm in presence of the dissipation of qubits. Roughly speaking it is an algorithm for evaluation of scientist quality. Is present researcher excellent? He introduces some factors (for example, h-factor and A-factor) for evaluation and calculates them under considering the author’s publications alone and with co-authors, in journals with high impact factor or without it and

so on.

The most participants were mathematicians and their talks about nonlinear dynamics and nonlinear differential equations were mathematical.

Professor Albert Luo and scientists from his school (Professors Yu Guo, USA, Xilin Fu, China, Siyuan Xing, USA) investigate periodic motions of the discontinuous dynamical systems. They have experimental equipment and may to compare analytical and numerical results with experimental ones. At the talk “On Bifurcation trees for period-1 motion to chaos in a periodically forced quadratic nonlinear oscillator with time delay” by Albert C.J.Luo and Siyuan Xing the corresponding stability and bifurcation analysis of periodic motions are completed through eigenvalue analysis. From the analytical prediction, numerical results of periodic motions in the time-delayed quadratic nonlinear systems are illustrated.

Professor James A. Yorke, USA, is very famous serious scientist. He is very picturesque man. He is elderly man of 73 years old with big grey beard. Every day including the day of his talking and the day of conference banquet he was wearing shorts and red long socks. He overturned his name badge and wrote by hand “Jim Yorke”. His talk “Chaos in higher dimensions” was very bright and rich. The title of one his poster was “It is the nature of chaotic systems to surprise”. He gave very obvious picture of three typical recurrent motions: chaotic, quasiperiodic, periodic, such as at Fig. 6.

In general the western scientists like free and picturesque style in their oral and written speeches. They devote scientific monographs to their wives, daughters and mothers. In his famous monograph [13] Professor Seydel names one section as “The Art and Science of Parameter Study”. He writes that “it is not always sufficient to rely on numerical methods only. To find all the branches good luck is also needed.” Now when we investigate the periodic motion stability, find bifurcation points by numerical parameter continuation method we always say that we have found so many different branches as good luck allowed us. Professor Enrique Ponce, Spain, is serious mathematician. At his talking he considered the false Fillippov systems, discontinuous piece-wise systems. His talk had the title “The boundary Focus in Planar Fillippov Systems: a

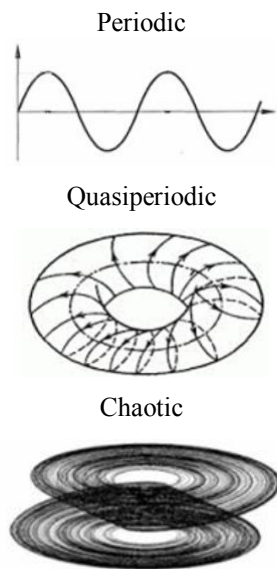


Fig. 6

Wolf in Sheep's Clothing". He often used the words: hunting, the wolf is coming and so on. Even the method of periodic solutions construction for nonlinear differential equations is called as shooting method for a long time.

At once the next day after conference we have received two invitations to submit our unpublished papers to American journals.

3. Examples of the phenomena unique for non-smooth discontinuous systems

Now we'll show the most spectacular results of our investigation. These results obviously show the phenomena unique for non-smooth discontinuous systems. Our vibroimpact system is exactly such one. These results were represented in our talk at Conference. Here we show our posters. All pictures illustrate the discontinuous bifurcation points on frequency-amplitude response that is under excitation frequency varying. At Fig.7 we show how two complex conjugate Floquet multipliers μ_1 and μ_2 are leaving the unit circle at discontinuous bifurcation point B . They are experiencing a discontinuous change by jump and accepting big values.

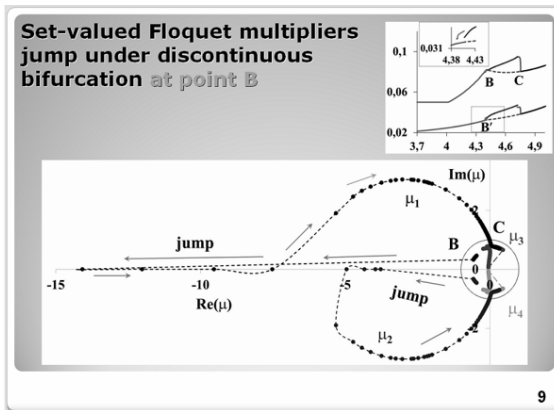


Fig. 7. Poster 9

At Fig. 8 one can see the jump of monodromy matrix and Floquet multipliers at this point B .

We have observed several different chattering regimes. Chatter (or rattle) is oscillatory regime with large period (nT -periodic regime where T is excitation period, n is sufficiently big number) with big impacts quantity per cycle. At non-smooth systems chattering regimes often lead to chaos. At Fig. 9 we depict the Poincare sections for such regimes under close excitation frequency.

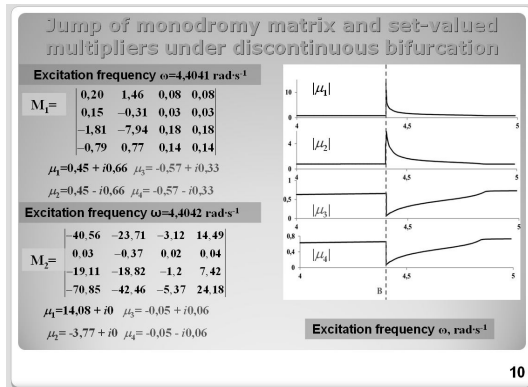


Fig. 8. Poster 10

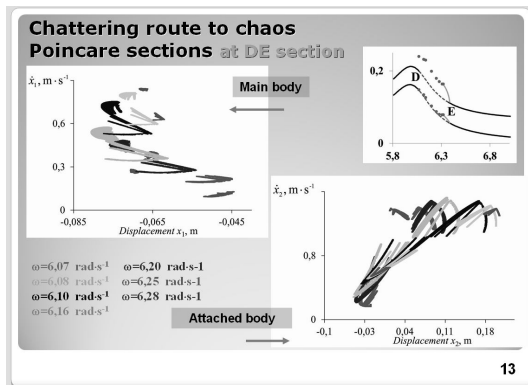


Fig. 9. Chattering route to chaos (Poster 13)

At Fig. 10 the point M of frequency-amplitude response is the turning point. At this point Floquet multiplier μ_1 passes through the unit circle over the $+1$ and moves along the positive real axis faraway. Its way under MN unstable regime is shown by flat points at Fig.10. Its velocity in motion along the axis is increasing and we see its appearance less and less often. The multiplier motion along positive real axis is demonstrated by Table at Fig.10. At point N we observe phenomenon unique to discontinuous system – discontinuous fold bifurcation. The discontinuous fold bifurcation connects a stable branch (solid line) to unstable branch (dotted line). Here set-valued Floquet multiplier μ_1 is returning (or is leaving) the unit circle by jump.

9. *Blaziejczyk-Okolewska B., Czolczynski K., Kapitaniak T.* Dynamics of a two-degree-of-freedom cantilever beam with impacts //Chaos, Solitons & Fractals. – 2009. – Т. 40. – №. 4. – С. 1991-2006.
10. *Peterka F.* An investigation of the motion of impact dampers, paper I, II, III //Strojnicku Casopis XXI, c. – 1971. – Т. 5.
11. *Bernardo M. et al.* Piecewise-smooth dynamical systems: theory and applications. – Springer Science & Business Media, 2008. – Т. 163.
12. *Leine R. I., Van Campen D. H.* Discontinuous bifurcations of periodic solutions //Mathematical and computer modelling. – 2002. – Т. 36. – №. 3. – С. 259-273.
13. *Seydel R.* Practical bifurcation and stability analysis. – Springer Science & Business Media, 2009. – Т. 5.
14. *Ivanov A. P.* Analysis of discontinuous bifurcations in nonsmooth dynamical systems //Regular and Chaotic Dynamics. – 2012. – Т. 17. – №. 3-4. – С. 293-306.
15. *Di Bernardo M. et al.* Bifurcations in nonsmooth dynamical systems //SIAM review. – 2008. – С. 629-701.
16. *Leine R. I., Van Campen D. H.* Discontinuous bifurcations of periodic solutions //Mathematical and computer modelling. – 2002. – Т. 36. – №. 3. – С. 259-273.
17. *Kowalczyk P. et al.* Two-parameter discontinuity-induced bifurcations of limit cycles: classification and open problems //International Journal of Bifurcation and Chaos. – 2006. – Т. 16. – №. 03. – С. 601-629.
18. *Brogliato B.* Nonsmooth mechanics: models, dynamics and control. – Springer Science & Business Media, 1999.
19. *Brogliato B.* Impacts in mechanical systems: analysis and modelling. – Springer Science & Business Media, 2000. – Т. 551.
20. *Alzate R.* Analysis and Application of Bifurcations in Systems with Impacts and Chattering : дис. – Università degli Studi di Napoli Federico II, 2008.
21. *Bernardo M. et al.* Piecewise-smooth dynamical systems: theory and applications. – Verlag, UK, 2007.
22. *Bazhenov V. A., Pogorelova O. S., Postnikova T. G.* Modification of the One-Parameter Numerical Continuation Method for Analysis of the Dynamics of Vibroimpact Systems //Strength of Materials. – 2014. – Т. 46. – №. 6. – С. 801-809.
23. *Bazhenov V. A., Lizunov P.P., Pogorelova O.S., Postnikova T.G., Otrasheskaia V.V.* Stability and Bifurcations Analysis for 2-DOF Vibroimpact System by Parameter Continuation Method. –of Applied Nonlinear Dynamics. – 2015. – Т. 4. – №. 4. (in press)
24. *Bazhenov V. A., Lizunov P.P., Pogorelova O.S., Postnikova T.G.* Numerical Bifurcation Analysis of Discontinuous 2-DOF Vibroimpact System. Part 2: Frequency-Amplitude response //Journal of Applied Nonlinear Dynamics. – 2016. (in press).
25. *Bazhenov V. A., Lizunov P.P., Pogorelova O.S., Postnikova T.G.* Bifurcation Points under 2-DOF Vibroimpact System Moving Numerical analysis by Parameter Continuation Method // Proceedings of the International Conference on Nonlinear Dynamics and Complexity (NDC2015). – 11-15 May 2015. – La Manga del Mar Menor, Spain.

Баженов В.А., Погорелова О.С., Постнікова Т.Г.

ДИНАМІЧНА ПОВЕДІНКА НЕЛІНІЙНОЇ НЕГЛАДКОЇ РОЗРИВНОЇ ВІБРОУДАРНОЇ СИСТЕМИ

Автори коротко описують стан проблеми вивчення динамічної поведінки розривних віброударних систем. В даний час дослідження таких систем розвиваються дуже швидко. Наводиться огляд світової наукової літератури про цю проблему. Міститься також інформація та опис Міжнародної Конференції про Нелінійну Динаміку Складних Структур. Автори показують власні найбільш ефектні результати, які демонструють явища унікальні для негладких систем, що описані диференціальними рівняннями з розривною правою частиною. Ці результати були отримані чисельним методом продовження розв'язку за параметром.

Ключові слова: віброударна система, розривна, конференція, методика продовження за параметром, нестійкість, матриця монодромії, мультиплікатори, точки біфуркації

ДИНАМИЧЕСКОЕ ПОВЕДЕНИЕ НЕЛИНЕЙНОЙ НЕГЛАДКОЙ РАЗРЫВНОЙ ВИБРОУДАРНОЙ СИСТЕМЫ

Баженов В.А., Погорелова О.С., Постникова Т.Г.

Авторы кратко описывают состояние проблемы изучения динамического поведения разрывных виброударных систем. В настоящее время исследования таких систем развивается очень быстро. Приводится обзор мировой научной литературы по этой проблеме. Дана информация и описание Международной Конференции по Нелинейной Динамике Сложных Структур. Авторы показывают свои результаты, демонстрирующие явления уникальные для негладких систем, описанных дифференциальными уравнениями с разрывной правой частью. Эти результаты были получены численным методом продолжения решения по параметру.

Ключевые слова: виброударная система, разрывная, методика продолжения по параметру, неустойчивость, матрица монодромии, мультипликаторы, точки бифуркации

UDC 539.3

Bazhenov V.A., Pogorelova O.S., Postnikova T.G. **Dynamic behaviour of nonlinear nonsmooth discontinuous vibroimpact system** // *Strength of Materials and Theory of Structures*. – 2015. – Issue. 95 (00). – P. 3 – 15.

The problem state about studying of discontinuous vibroimpact systems dynamical behaviour is described, information about an International Conference on Nonlinear Dynamics Complexity (NDC2015) is given.

Table 0. Fig. 10. Ref. 25

УДК 539.3

Баженов В.А., Погорелова О.С., Постнікова Т.Г. **Динамічна поведінка нелінійної негладкої розривної віброударної системи** // *Опір матеріалів і теорія споруд*. – 2015. – Вип. 95 (00). – С. 3 – 15.

Розглядається стан проблеми вивчення динамічної поведінки розривних динамічних систем, наводиться інформація щодо Міжнародної Конференції про Нелінійну Динаміку Складних Структур (NDC2015).

УДК 539.3

Баженов В.А., Погорелова О.С., Постникова Т.Г. **Динамическое поведение нелинейной негладкой разрывной виброударной системы** // *Сопротивление материалов и теория сооружений*. – 2015. – Вып. 95 (00). – С. 3 – 15.

Рассматривается состояние проблемы изучения динамического поведения разрывных виброударных систем, приводится информация о Международной Конференции по Нелинейной Динамике Сложных Структур (NDC2015).

Автор (вчена ступень, вчене звання, посада): доктор технічних наук, професор, академік Національної академії педагогічних наук України, зав. кафедрою будівельної механіки КНУБА, директор НДІ будівельної механіки КНУБА БАЖЕНОВ Віктор Андрійович
Адреса робоча: 03680 Україна, м. Київ, Повітрофлотський проспект 31, Київський національний університет будівництва і архітектури, БАЖЕНОВУ Віктору Андрійовичу
Адреса домашня: 01001 м. Київ, вул. Заньковецької, б. 5/2, кв. 31. БАЖЕНОВУ Віктору Андрійовичу.
Роб. тел. +38(044) 245-48-29.

Автор (вчена ступень, вчене звання, посада): кандидат фізико-математичних наук, старший науковий співробітник, провідний науковий співробітник НДІ будівельної механіки ПОГОРЕЛОВА Ольга Семенівна
Адреса робоча: 03680 Україна, м. Київ, Повітрофлотський проспект 31, Київський національний університет будівництва і архітектури, ПОГОРЕЛОВІЙ Ользі Семенівні.
Адреса домашня: 02147 Україна, м. Київ, вул. Русанівська Набережна 18, кв. 61, ПОГОРЕЛОВІЙ Ользі Семенівні
Роб. тел. +38(044) 245-48-29;
мобільний тел.: +38(067) 606-03-00;
дом. тел.: +38(044) 517-25-08.
E-mail – pogos13@ukr.net

Автор (вчена ступень, вчене звання, посада): кандидат технічних наук, старший науковий співробітник, старший науковий співробітник НДІ будівельної механіки ПОСТНІКОВА Тетяна Георгіївна
Адреса робоча: 03680 Україна, м. Київ, Повітрофлотський проспект 31, Київський національний університет будівництва і архітектури, ПОСТНІКОВІЙ Тетяні Георгіївні.
Адреса домашня: 04210, Україна, м. Київ, пр. Героїв Сталінграду 24, кв. 17, ПОСТНІКОВІЙ Тетяні Георгіївні
Роб. тел. +38(044) 245-48-29;
мобільний тел.: +38(050) 353-47-19;
дом. тел.: +38(044) 411-46-56.
E-mail – posttan@ukr.net