Analysis of the predictive properties of Brown's model in the extended domain of the internal parameter

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Summary. The paper analyses the predictive properties of Brown's adaptive model in the extended domain of internal parameters, which relates to the class of problems in parametric synthesis of forecast models, namely: evaluating the stability of model predictive properties to variation of internal parameters by searching for forecast robustness domains. The approach suggested is illustrated by an example.

Key words: Brown's model, exponential smoothing, parametric synthesis, forecast robustness.

INTRODUCTION

Among the key functions of systems for controlling social and economic processes according to [1] are forecasting and process planning. Implementing this function without advanced forecasting methods is impossible, and any attempts to manage without them in current conditions are foredoomed, at least, to a financial failure. Underestimating the importance of forecasting and the quality of forecasts downplays the competitive advantages of enterprises and organizations. This makes forecasting one of the key tasks in controlling social and economic processes. Proper usage of predictive models, a clear understanding of their internal workings, and a knowledge of the limits of model adequacy are the necessary conditions for quality and well-grounded managerial decisions, and consequently, for effective management as a whole.

This paper analyses the predictive properties of Brown's adaptive model in the extended domain of internal parameters, which relates to the class of problems in parametric synthesis of forecast models.

REVIEW OF PUBLICATIONS

R. Brown suggested his predictive model [2] or exponential smoothing model in the late '50s of the last century and found application in tens of engineering's tasks [3-8]. His concept was to use the exponential average value of a stationary time series:

$$F_t = \alpha A_{t-1} + \alpha(1-\alpha) A_{t-2} + +\alpha(1-\alpha)^{n-1} A_{t-n} = \sum_{i=1}^{n} \alpha(1-\alpha)^{i-1} A_{t-i}, \tag{1}$$

for short-term forecasts, where F_t is forecast at point of time t (exponential mean), $A_{t-1}, A_{t-2}, ..., A_{t-n}$ are series values at respective time points, n is time series length, α is smoothing factor (a constant).

Practical application of Brown's model requires solving the model parametric setting problem, i.e. substantiate the choice of smoothing factor α. Many publications have dealt with the problem of choosing this Brown's model factor, e.g. [9-15]; however, to date there is no single approach to this.

The classical range of admissible values of the smoothing factor is the interval $\alpha \in [0, 1]$. This range is logically conditioned by the necessity to ensure convergence of the series of weight coefficients in formula (1)

$$\{a_k\}_{k=1}^{n} = \alpha,\ \alpha(1-\alpha),\ \dots,\ \alpha(1-\alpha)^{n-1} \quad (2)$$

to unit.

At the turn of the Millennium, S.G. Svetun'kov in his studies, e.g. [10], demonstrated that the classical range $\alpha \in [0, 1]$ could be extended to $\alpha \in [0, 2]$ without violating the condition of convergence of weight coefficients series (2) to unit. In this case, series (2) changes from a fixed-sign one in the interval $\alpha \in [0, 1]$ to a variable-sign one in the interval $\alpha \in (1, 2]$.

Set $\alpha \in (1, 2]$ of the internal factor of Brown's predictive model is known as the 'out-of-limit' one [16-18] or Svetun'kov's set [19].

Let set K_c be a classical admissible set, set K_{out} be an out-of-limit admissible set, and set $K_{ext} = K_c \cup K_{out}$ be an extended admissible set of smoothing factor α:

$$\begin{cases} K_c = \{\alpha :\ 0 \le \alpha \le 1\}, \\ K_{out} = \{\alpha :\ 1 < \alpha \le 2\}, \\ K_{ext} = \{\alpha :\ 0 \le \alpha \le 2\}. \end{cases} \quad (3)$$

PROBLEM STATEMENT

The objective of this study is investigating the predictive properties of Brown's model on an extended set K_{ext} of internal factor α, and ensuring stability of model predictive properties to variations of internal factors by searching for forecast robustness domains.

MAIN PART

Let us investigate the behaviour of the sum of series (2) with an increasing number of its terms n on extended set K_{ext} of smoothing factor α:

$$S_n = 1 - (1-\alpha)^n. \quad (4)$$

Fig. 1 shows dependence $S_n(\alpha, n)$ according to (4).

From Fig. 1, it is obvious that the sum of coefficients in (1) is not equal to unit in all cases. This means that Brown's model uses strictly speaking not the exponential average as a forecast, but the exponential weighted value of the initial series.

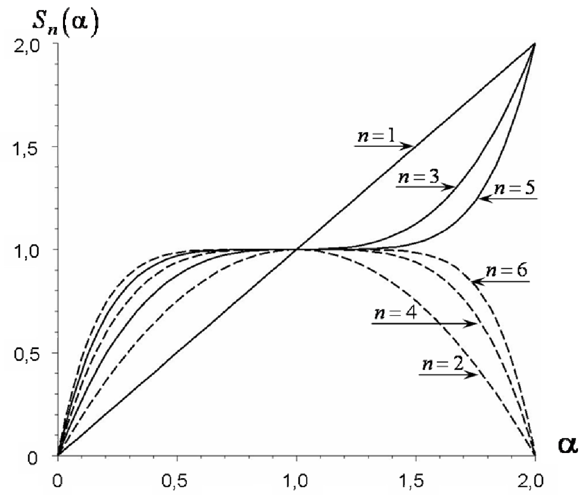

Fig. 1. Sum of series of Brown's model weight coefficients vs. smoothing factor α and number of series elements n on extended set K_{ext}

The closeness of the forecast to the exponential average can be evaluated analytically. For this, let us transform dependence (4) by mirror imaging a group of growing branches with respect to a unit level.

Fig. 2 shows dependence

$$S'_n = 1 - |1-\alpha|^n. \quad (5)$$

Fig. 2, besides showing dependence $S'_n(\alpha,n)$, shows a plane at level $1 - 0{,}01\lambda = 0.95$, where λ is measure of closeness to the exponential average value. It intercepts the domain of parameters in plane (α,n), within which the predictive value is close to the exponential average one by less than λ percent.

The boundaries of this domain can be found from relationship

$$1 - |1-\alpha|^n \ge 1 - 0{,}01\lambda. \quad (6)$$

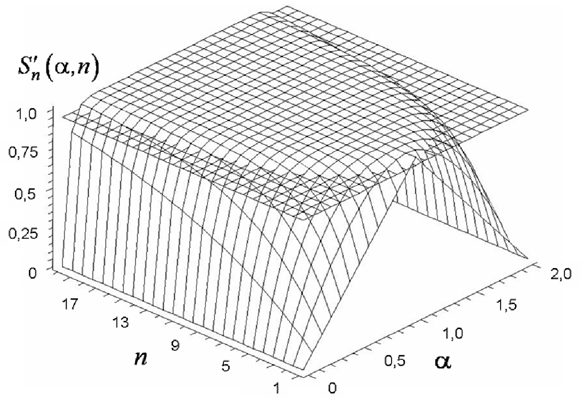

Fig. 2. Transformed sum of Brown's model weight coefficients vs. smoothing factor α and number of series elements n on extended set K_{ext}

Hence,

$$\left|1-\alpha\right|^n \le 0{,}01\lambda, \qquad (7)$$

or finally,

$$1-\left(0{,}01\lambda\right)^{1/n} \le \alpha \le 1+\left(0{,}01\lambda\right)^{1/n} \quad . \qquad (8)$$

The domain satisfying (8) is shown in Fig.3.

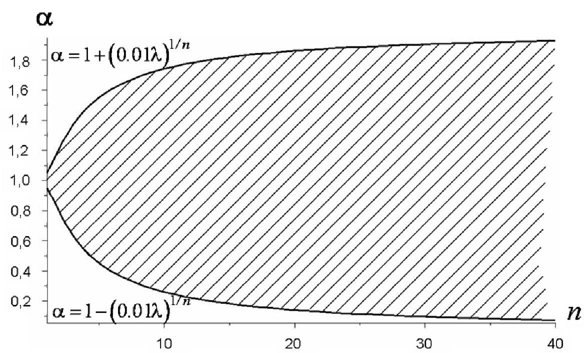

Fig. 3. Domain in plane of factors (α,n) ensuring closeness of Brown's model forecast to the average exponential value of n series elements by less than λ percent ($\lambda = 5$)

The parametric synthesis problem can be solved analytically only 'retrospectively', i.e. for time points $(t-1)$, $(t-2)$ and earlier ones [14]. This requires solving retrospective equations of the kind:

$$F_{t-1}(\alpha) = \sum_{i=1}^{n-1} \alpha(1-\alpha)^{i-1} A_{t-i-1} = A_{t-1}. \qquad (9)$$

Let us consider a situation when an equation of the (9) kind has been formed for one time point $(t-1)$ and has more than one real root on extended admissible set $K_{ext} = \{\alpha:\ 0 \le \alpha \le 2\}$. This means that there are real $\alpha_1, \alpha_2, ..., \alpha_j$, $j \ge 2$, which being the roots of retrospective equation (9), would ensure an accurate forecast at point of time $(t-1)$.

Hence, one faces the problem of a well-grounded choice among $\alpha_1, \alpha_2, ..., \alpha_j$ of smoothing factor α values for forecasting at time point t.

Obviously, as to their retrospective accuracy, all values $\alpha_1, \alpha_2, ..., \alpha_j$ are equivalent by virtue of the concept of retrospective analysis, i.e. ensuring absolute accuracy for past time points with respect to t.

In this situation, the criteria for choosing smoothing factor α can be sensitivity and robustness of forecasts obtained for $\alpha_1, \alpha_2, ..., \alpha_j$. The following method is suggested for choosing smoothing factor α for the above-stated conditions (Fig. 4).

Stage 1. *Forming retrospective equation* of the kind (9) for time point $(t-1)$ and a sampling length of n elements.

Stage 2. *Searching for the real roots of retrospective equation* of the kind (9) by using applied mathematical software packages (for instance, Maple) or the graphical method.

If no real roots exist on the extended admissible set $K_{ext} = \{\alpha:\ 0 \le \alpha \le 2\}$, then Brown's model (1) is inapplicable for predicting the series being investigated and requires a structural complication.

If on set K_{ext} there exists one real root, it shall be accepted as the value of smoothing factor α for forecasting at time point t. Sensitivity and robustness of the forecast can be evaluated according to the following stages, though in this case they cannot be the criteria for parametric setting of the model.

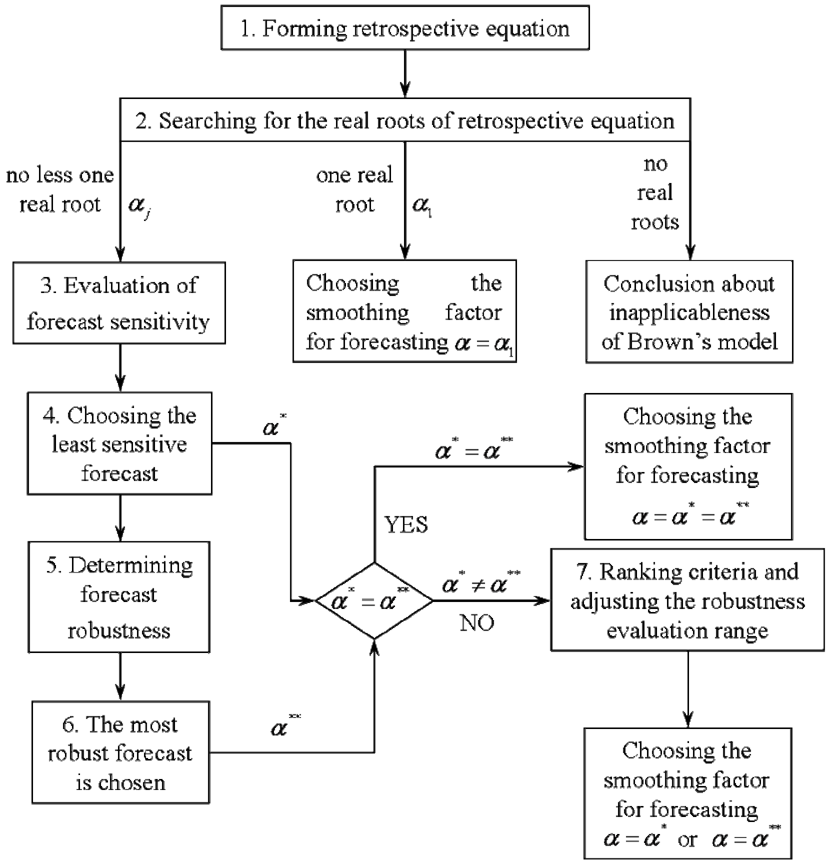

Fig. 4. Method for choosing smoothing factor α by the criteria of sensitivity and robustness of retrospective forecasts

Stage 3. *Evaluation of forecast sensitivity* is suggested to be done by calculating the sensitivity equal to the module of the derivative of forecast function $F'_{t-1}(\alpha)$ in points $\alpha = \alpha_1,\ \alpha = \alpha_2,\ ...,\ \alpha = \alpha_j$, $j \geq 2$, where $\alpha_1, \alpha_2, ..., \alpha_j$ are real roots of retrospective equation (9).

Stage 4. *Choosing the least sensitive forecast*. This is done by solving the optimization problem:

$$\alpha = \alpha^* : \ \left|F'_{t-1}\left(\alpha^*\right)\right| = \min_i \left|F'_{t-1}(\alpha_i)\right|,\ i = \overline{1, j}\,. \quad (10)$$

Value $\alpha = \alpha^*$ ensures minimal forecast sensitivity to small variations of smoothing factor α in the vicinity of α^*.

Stage 5. *Determining forecast robustness*. Forecast robustness can be evaluated graphically by showing the sensitivity of the forecast relative error to smoothing factor α variations.

For this, we shall substitute

$$\alpha = \alpha_i + \Delta\alpha_i,\ i = \overline{1, j}, \quad (11)$$

where α_i are real roots of equation (9), into the expression for the forecast relative error ε_F

$$\varepsilon_F = \frac{F_{t-1}(\alpha) - A_{t-1}}{A_{t-1}} \cdot 100\,. \quad (12)$$

Systematic error $\Delta\alpha_i$ with respect to real root α_i can be expressed through the relative error of choosing smoothing factor α:

$$\Delta\alpha_i = 0{,}01\alpha_i\varepsilon_\alpha, \quad (13)$$

where ε_α is relative error of choosing smoothing factor α in percent.

With account of the symmetry of the function of the sum of weight coefficients (4) in

Brown's model on classical K_c and out-of-limit admissible set K_{out}, expression (13) shall be used with $\alpha_i \in [0, 1]$, and with $\alpha_i \in [0, 1]$ $\Delta\alpha_i$ shall take the form:

$$\Delta\alpha_i = 0{,}01(2-\alpha_i)\varepsilon_\alpha . \qquad (14)$$

Making all the substitutions (12) yields

$$\varepsilon_F = \frac{100}{A_{t-1}} \times \sum_{i=1}^{n-1} \alpha_i (1+0{,}01\varepsilon_\alpha)(1-\alpha_i - 0{,}01\alpha_i\varepsilon_\alpha)^{i-1} A_{t-i-1} - 100, \ \alpha_i \in [0, 1] \qquad (15)$$

and

$$\varepsilon_F = \frac{100}{A_{t-1}} \times \sum_{i=1}^{n-1} (\alpha_i + 0{,}01(2-\alpha_i)\varepsilon_\alpha) \times (1-0{,}01(2-\alpha_i)\varepsilon_\alpha)^{i-1} A_{t-i-1} - 100, \ \alpha_i \in (1, 2]. \qquad (16)$$

If dependencies (15) and (16) for all real roots of retrospective equation (9) with a total number of j shall be shown in a single plane of parameters $(\varepsilon_F, \varepsilon_\alpha)$, then one can easily evaluate the degree of robustness of forecasts obtained for different α (Fig. 5).

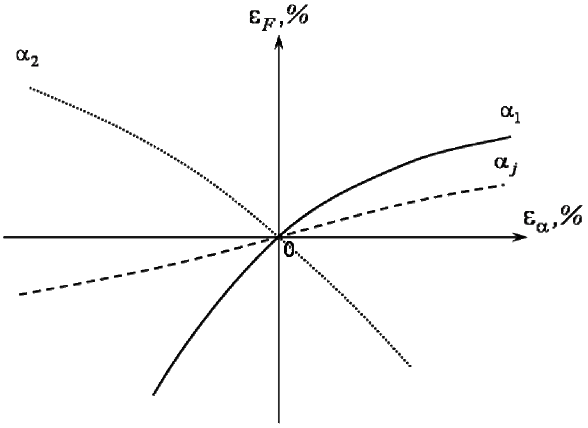

Fig. 5. Sensitivity of forecast relative error ε_F to variation of smoothing factor α by ε_α percent with respect to retrospective equation roots

All the curves in Fig. 5 corresponding to real roots of retrospective equation (9) pass through the origin of coordinates because the forecast relative error at $\alpha = \alpha_i$, $i = \overline{1, j}$ equals zero.

The closer the curve approaches the X-axis the less sensitive is the forecast to variation of α, and hence, it possesses better robustness.

Analytically, it is suggested to be evaluated by an inverse of the module of the definite integral of function $\varepsilon_F(\varepsilon_\alpha)$ over a concrete interval. Let it be called the robustness parameter:

$$r_i^{(\beta)} = \frac{1}{\int_{-\beta}^{\beta} \left|\varepsilon_{F_i}(\varepsilon_\alpha)\right| d\varepsilon_\alpha}, \qquad (17)$$

where $r_i^{(\beta)}$ is robustness parameter for the i-th forecast in the range $(-\beta;\beta)$, $\varepsilon_{F_i}(\varepsilon_\alpha)$ is the analytical dependence of the error of the i-th forecast on the error of choosing the smoothing factor.

Obviously, $r_i^{(\beta)} \in (0, \infty)$. Small values of the robustness parameter mean significant sensitivity or forecast instability to smoothing factor α variations. Big values of the robustness parameter mean that, over the whole interval $(-\beta;\beta)$, the sensitivity curve in Fig. 5 is in close proximity to the X-axis, ensuring thereby insensitivity or stability of forecast quality to smoothing factor α variations.

Stage 6. *The most robust forecast is chosen* by solving optimization problem:

$$\alpha = \alpha^{**} : \ r_i^{(\beta)}(\alpha^{**}) = \max_i r_i^{(\beta)}, \ i = \overline{1, j}. \qquad (18)$$

In case of matching optimal values of α^* and α^{**} found by sensitivity and robustness criteria, respectively, choosing the smoothing factor for forecasting for the next time period $\alpha = \alpha^* = \alpha^{**}$ seems well-grounded.

Stage 7. *Ranking criteria and adjusting the robustness evaluation range*. Fig. 6 shows the case when $\alpha^* \neq \alpha^{**}$, and poses the problem of ranking sensitivity and robustness criteria.

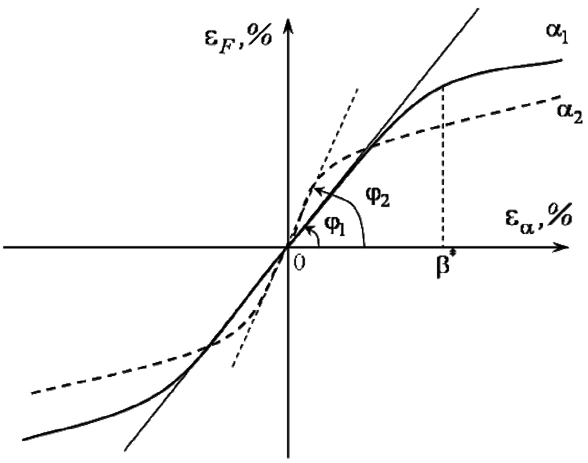

Fig. 6. Arrangement of sensitivity curves of forecast errors for $\alpha^* \neq \alpha^{**}$

Sensitivity is a moment or differential estimate characterizing the sensitivity curve slope in point $\alpha = \alpha_i$ or $\varepsilon_\alpha = 0$ (in Fig. 6, $\varphi_1 < \varphi_2$). Robustness is an integral estimate characterizing the area under the sensitivity curve (in Fig. 6, $r_1^{(\beta)} > r_2^{(\beta)}$ for $\beta < \beta^*$ and $r_1^{(\beta)} < r_2^{(\beta)}$ for $\beta > \beta^*$).

Hence, the researcher is forced to determine one's subjective preference in regard to criteria or determine such a range $(-\beta;\beta)$, for which the solutions of the optimization problem for two criteria match.

Example. As an example, let us consider a series of climate data from the weather conditions archive (http://meteo.infospace.ru) namely: sea level atmospheric pressure P_0 recorded from 26.11.1998 to 2.02.1999 by the Kharkiv Weather Station daily at 12:00 local time (Fig. 7).

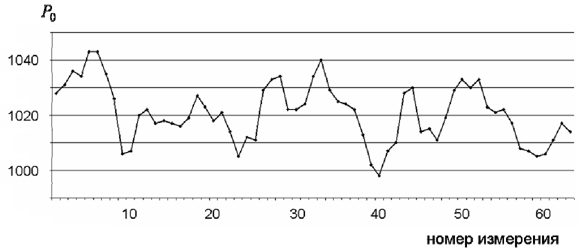

Fig. 7. Climate data series

Let us apply the suggested method of choosing α for sampling from the 47th to the 57th series elements.

Stage 1. The following retrospective equation is formed:

$$\begin{aligned} F_{12} &= 1011\alpha^{11} - 11129\alpha^{10} + 55695\alpha^9 - \\ &-167269\alpha^8 + 334979\alpha^7 - 469696\alpha^6 + \\ &+470527\alpha^5 - 336738\alpha^4 + 168687\alpha^3 - \\ &-56305\alpha^2 + 11246\alpha = 1007. \end{aligned} \tag{19}$$

Stage 2. The real roots of the equation are located on the extended admissible set of smoothing factor α:

$$\alpha_1 = 0{,}3439;\ \alpha_2 = 1{,}1192;\ \alpha_3 = 1{,}5900. \tag{20}$$

Stage 3. Let us calculate the derivative

$$\begin{aligned} F'_{12} = \frac{dF_{12}}{d\alpha} &= 11121\alpha^{10} - 111290\alpha^9 + \\ &+501255\alpha^8 - 1338152\alpha^7 + 2344853\alpha^6 - \\ &-2818176\alpha^5 + 2352635\alpha^4 - 1346952\alpha^3 + \\ &+506061\alpha^2 - 112610\alpha + 11246 \end{aligned} \tag{21}$$

in points (20):

$$\begin{cases} F'_{12}(\alpha_1) = 145{,}6646, \\ F'_{12}(\alpha_2) = -7{,}7603, \\ F'_{12}(\alpha_3) = 48{,}0280. \end{cases} \tag{22}$$

Stage 4. The smaller by module derivative indicates the least sensitive forecast obtained with smoothing factor $\alpha^* = \alpha_2 = 1{,}1192$ (Fig. 8).

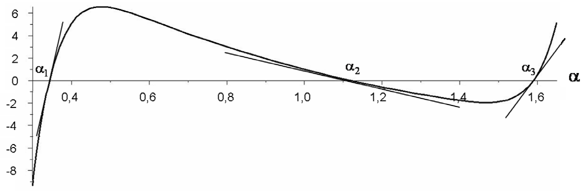

Fig. 8. Real roots of retrospective equation (19)

Stage 5. Graphical evaluation of forecast robustness is shown in Fig. 9.

Fig. 9 shows that the forecast obtained for $\alpha = \alpha_2$ is the least sensitive to variation of smoothing factor α, and hence, is more robust. Note that the forecast obtained for $\alpha = \alpha_1$ has the worst robustness of the three ones, though when the classical admissible set $K_c = \{\alpha : 0 \le \alpha \le 1\}$ is used it is the only admissible one.

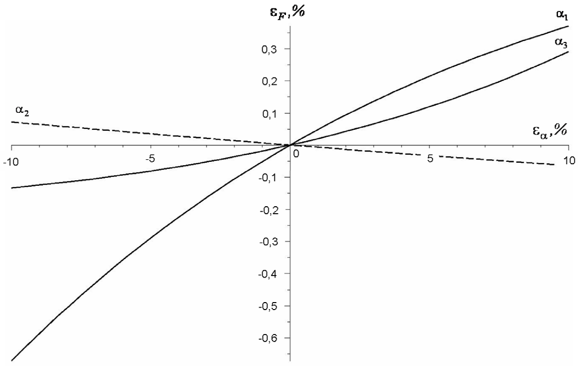

Fig. 9. Graphical evaluation of forecast robustness

Let us determine the robustness parameters for three forecasts for $\beta = 10\%$:

$$\begin{cases} r_1^{(10)} = \dfrac{1}{\int\limits_{-10}^{10} \left|\varepsilon_{F1}(\varepsilon_\alpha)\right| d\varepsilon_\alpha} = 0{,}1964, \\ r_2^{(10)} = 1{,}4731,\ r_3^{(10)} = 0{,}4907. \end{cases} \quad ; \qquad (23)$$

Stage 6. The most robust forecast corresponds to $\alpha^{**} = \alpha_2 = 1{,}1192$.

Stage 7. In this case, criteria ranking and adjustment of the robustness evaluation range is not required because $\alpha^* = \alpha^{**} = 1{,}1192$. This value of α will be used for forecasting for the next time point. The forecast shall be calculated for two samples with a length of 11 and 12 series elements, having compared them for relative accuracy ($\varepsilon_i^{(11)}$ and $\varepsilon_i^{(12)}$). The simulation results are shown in Table 1.

As Table 1 shows, choosing smoothing factor $\alpha = \alpha^* = \alpha^{**} = 1{,}1192$ ensures not only the robustness of the retrospective forecast to variation of α, but also the robustness of the current forecast to variations in sample length n.

Table 1. Retrospective analysis of climate data and evaluating the sensitivity and robustness of forecasts

Series elements	47-57	47-57	47-57
i	1	2	3
α_i	0,3439	1,1192	1,5900
$F'(\alpha_i)$	145,6646	**−7,7603**	48,0280
$r_i^{(10)}$	0,1964	**1,4731**	0,4907
$\varepsilon_i^{(11)}$	-0,1369	**0,1990**	0,6816
$\varepsilon_i^{(12)}$	0,1986	**0,1990**	0,1990

CONCLUSIONS

Using an extended admissible set of smoothing factor α in Brown's model requires additional analysis of the properties of the series and the model per se because the algebraic properties of series (2) of the model weight coefficients are different on the classical admissible set K_c and the out-of-limit admissible set K_{out}. Reducing the process of parameter setting of Brown's model to simple "smoothing factor choosing" often unduly simplifies forecasting and results in loss of model adequacy, and hence, forecast accuracy. A method has been suggested for choosing smoothing factor α by the criteria of sensitivity and robustness of retrospective forecasts. It allows determining the setting parameters of Brown's model ensuring maximum stability of forecasts to variations of model internal parameters. The method suggested is illustrated by an example using a set of real climate data.

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АНАЛИЗ ПРОГНОЗНЫХ СВОЙСТВ МОДЕЛИ БРАУНА В РАСШИРЕННОЙ ОБЛАСТИ ВНУТРЕННЕГО ПАРАМЕТРА

Аннотация. Публикация посвящена анализу прогнозных свойств адаптивной модели Брауна в расширенной области внутренних параметров, относящемуся к классу задач параметрического синтеза прогнозных моделей, а именно оценке устойчивости прогнозных свойств модели к изменению внутренних параметров путем поиска областей робастности прогнозных оценок. Предложенный подход проиллюстрирован примером.

Ключевые слова: модель Брауна, экспоненциальное сглаживание, параметрический синтез прогнозной модели, робастность прогнозных оценок.