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A METHODOLOGY OF DETERMINING OF PARAMETER *J** IN DISCRETE MODELS OF FINITE ELEMENT METHOD

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Based on the method of reactions, a technique for determining of the parameter J^* by the method of subdomain moving in discrete models of finite element method (FEM) has been developed. A number of test problems solved. The obtained results confirm the effectiveness of the technique.

Keywords: fracture mechanic, semi-analytical finite element method, J^* parameter, method of reaction.

Introduction. The solution of fracture mechanics problems of determination of the stress-strain state and the bearing capacity of the bodies with cracks is devoted in a large number of scientific publications as a fundamental nature, for example, [3, 5, 9] and those which containing solutions of partial questions. Taking into account the necessity of modeling of the behavior of complex form bodies with cracks, the essential heterogeneity of the distribution of stresses and displacements in the crack tip vicinity and the possibility of nonlinear deformations of plasticity the finite element method (FEM) is the most effective in solving of these problems. The use of FEM requires the development of special methods for calculating of the criterion parameters of the fracture mechanics - stress intensity factor (CIF), Cherepanov-Rice *J*-integral and also the intensity of energy of the formation of new surfaces of the crack (Griffiths *G*-criterion). Among the methods for determining of fracture mechanics parameters on the basis of FEM energy approaches are most widely used. Authors of this article conducted numerous studies that showed the effectiveness of the method of reactions in the implementation of the energy approach. However, the question of determining of the Cherepanov-Rice's *J*-integral [1, 2, 4] is considered in these studies. It is well known that in case of the presence of volume forces of different nature, the *J*-integral cannot be used to evaluate the crack resistance. The Griffith parameter (*G*) can be used as a parameter of fracture mechanics in this case. The method of determining of *G* using the reaction method is described in [8]. For the case of volume forces caused by the effect of temperature influences, the parameter J^* is proposed as a parameter of fracture [5, 6, 9]. A new approach based on the reaction method for determine the parameter J^* is developed in this paper.

1. Technique for *J evaluation.** In accordance to [5, 6, 9] under termoforce loading the value of parameter J^* is calculated using of formula:

$$J^*=J+\int_V \alpha\sigma_{ij}\varepsilon_{ij}\frac{\partial T}{\partial x}dV. \qquad (1)$$

The first component − the Cherepanov-Rice integral J − is determined by contour integration S covering the domain V:

$$J=\int_S\left(Wn_t-\sigma^{ij}\frac{\partial u}{\partial x}n_j\right)dS. \qquad (2)$$

The second component is determined by integration the internal volume of the region V.

The generalized version of the method of reactions, which based on the calculation of volume invariant integrals, is adopted as a basis for the determination of J^* in this paper. This method is similar to the approach outlined in [8].

In contrast to the calculation procedure of G, which involves changing the length of the crack l_{cr}, when calculating the parameter J^* the length of the crack is fixed. The substituent position of the region V changes to one finite element in relation to the crack tip (Fig. 1).

We denote as $\{u\}_I$, $\{R\}_I$ and $\{u\}_{II}$, $\{R\}_{II}$ vectors of displacements and node reactions of the subdomain V in positions "1" and "2", which are differ with the location of subdomain in relation to the crack front. Each of these vectors contains of $3N$ components, representing values of displacements and node reactions in three directions respectively and has the form:

$$\{u\}_I^T=\{(u_{1'})_1\ \ (u_{2'})_1\ \ (u_{3'})_1\ \ \ldots\ \ (u_{k'})_n\ \ \ldots\ \ (u_{3'})_{N-1}\ \ (u_{1'})_N\ \ (u_{2'})_N\ \ (u_{3'})_N\},$$

where the indices k'= 1, 2, 3 represent the directions of displacement (or node reaction), and the indices n = 1, 2, ... N are the numbers of nodes within each of the subdomain.

We allocate the axis $z^{2'}$ of the global coordinate system of a finite element model along the surface of the crack (perpendicular to the front and in the assumed direction of crack propagation). Then, under the condition of a regular finite element mesh in the direction of $z^{2'}$, the definition formula for J^* in discrete models acquires the following form [1]:

$$J^*=\frac{1}{2\Delta z^{2'}}\left(\{u\}_{II}^T\{R\}_{II}-\{u\}_I^T\{R\}_I\right)-\frac{1}{2\Delta z^{2'}}\left(\{u\}_{II}^T-\{u\}_I^T\right)\left(\{R\}_I+\{R\}_{II}\right). \qquad (3)$$

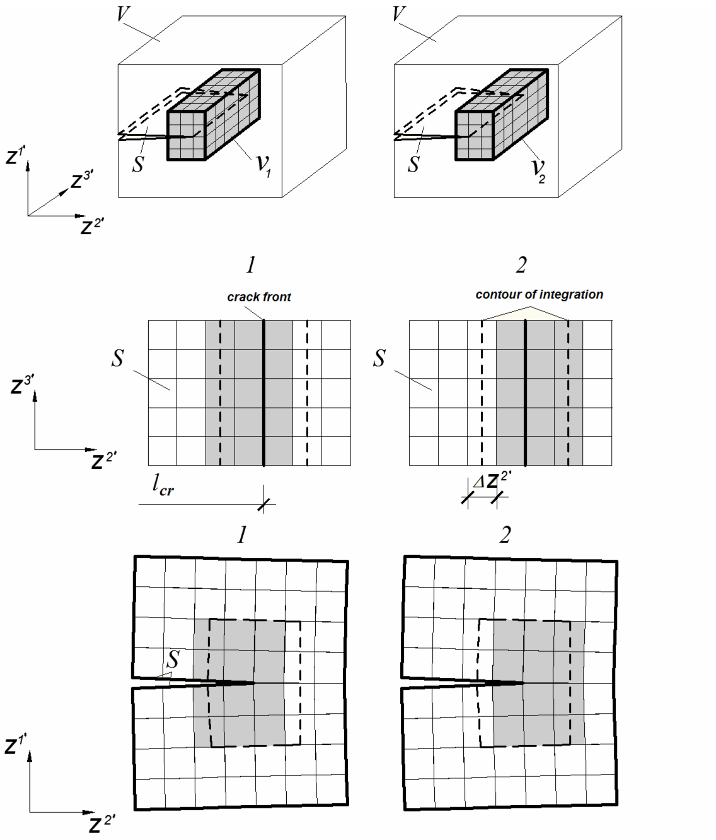


Fig. 1. Sheme of subdomain moving for calculation of parameter J^*

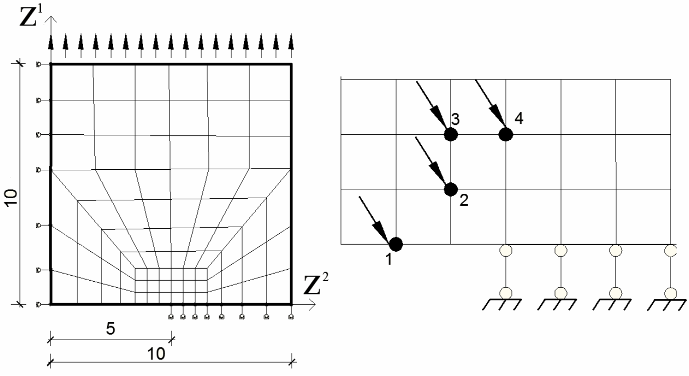
After multiplying of the nodes reactions to the corresponding displacements and adding such components, formula (3) will look like:

$$J^* = \frac{1}{2\Delta z^{2'}}\left(\{u\}_I^T\{R\}_{II} - \{u\}_{II}^T\{R\}_I\right). \tag{4}$$

It is not difficult to prove that in the absence of volume forces, this formula is identical to the formula being obtained directly from the expression of the Cherepanov-Rice integral for its calculation by the reaction method [1].

Verification of the developed technique is carried out on the problem of stretching of the plate with a central crack (Fig. 2, *a*). The calculation was carried out both under the action of the surface load and taking into consideration of the volume forces that additionally applied to the plate (the forces of 2, 3, 4, 5 *kg*, directed along Z^1 and Z^2 in the nodes 1, 2, 3, 4,

Fig. 2, *b*, q=1 *kg/sm*, Young's module E=1 *kg/sm*^{2}, Poison ratio ν=0.3). The obtained results were compared with the values of the Griffiths parameter, which was calculated according to the method described in [8].



a *b*

Fig. 2. Square plate with a central crack

The results of calculation, which presented in Table 1, indicate that the value of G and J^* differ by less than 1%.

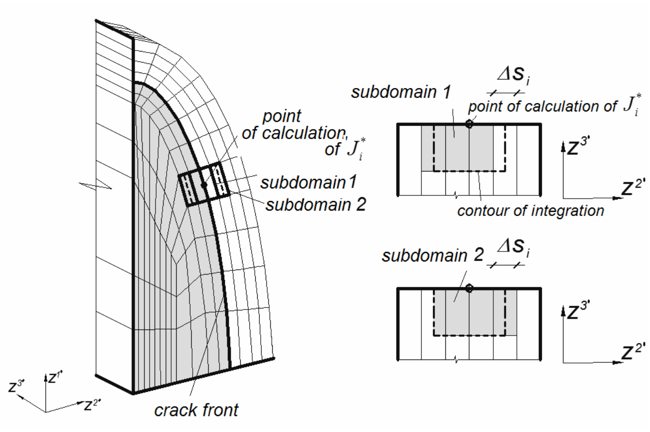
Table 1

Type of load	Fracture parameter		δ , %
	G	*J**	
Surface load	24.52	24.35	0.7
Volume loda	261.2	260.05	0.4

2. Implementation of the technique in the semi-analytical finite element method (SFEM). When the numerical realization of the formula (4) made using of SFEM, the points of calculation of J^* in bodies with crosscut cracks are located in the centers of the lines of the crack front, which are lateral faces of two finite element adjacent to the front (Fig. 3, *a*). Separate volumes of integration and their corresponding sub-domain determine for each such point (Fig. 3, *a*, *b*). The formula for calculation of the integral for a separate point in the front acquires the following form:

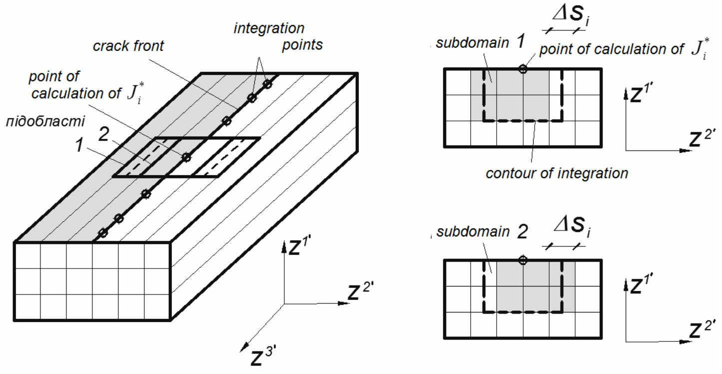
$$J_i^* = \frac{1}{2\Delta s_i}\left(\{u\}_I^T\{R\}_{II} - \{u\}_{II}^T\{R\}_I\right). \tag{5}$$

In bodies with longitudinal cracks the points of calculation of J^* will coincide with the integration points located along the front of the crack (Fig. 4).



a *b*

Fig. 3. Calculation of *J** in bodies with crosscut crack using of SFEM



a *b*

Fig. 4. Calculation of *J** in bodies with longitudinal crack using of SFEM

Approbation of the developed technique in spatial bodies was carried out on the test problem of the definition of *G* in a prismatic body with a lateral incision of zero thickness under the action of force load (Fig. 5). The results of the calculation of J^* are almost coincide with values of *G* obtained in [8] and are well consistent with the results of the calculation of *G(K)* based on the values of CIF being calculated by the direct method [7] (Fig. 5).

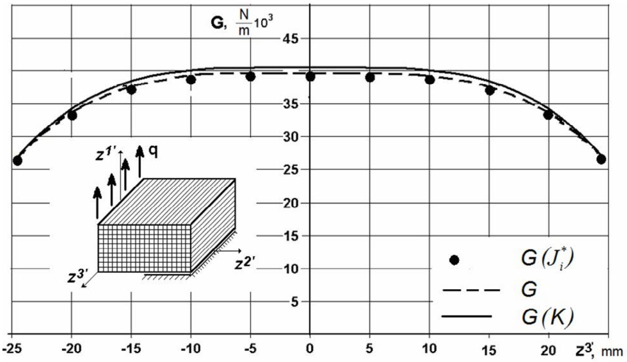


Fig. 5. Distrubution of Griffiths parameter G along the front of lateral incision in a prismatic body

3. Reliability of results under temperature load. A further study of reliability was carried out for the case of temperature load. A test problem of deformation of a long thick-walled cylinder with a crack was considered at first (Fig. 6).

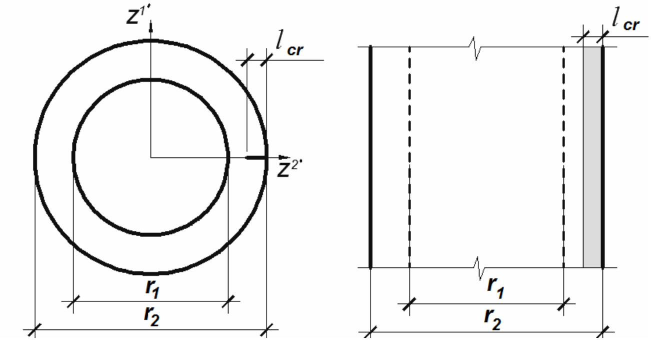


Fig. 6. Geometric scheme of a thick-walled cylinder with a crack

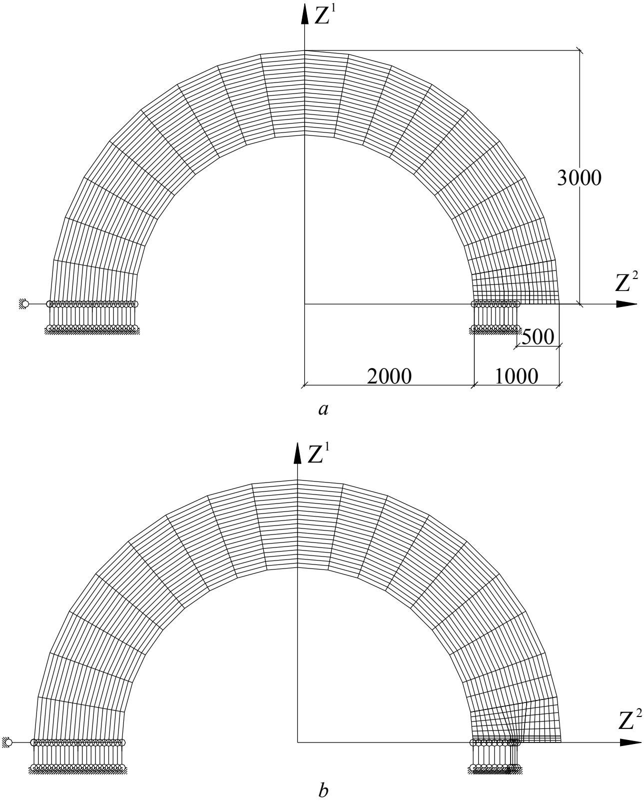
The material of cylinder is steel “38ХН3МФА”, Young's module E=210 GPa, Poison ratio ν=0.3, coefficient of linear expansion $\alpha = 13.5 \cdot 10^{-6}\ \deg^{-1}$. The radial distribution of temperature is described by the logarithmic law:

$$T = T_1 + (T_2 - T_1)\frac{\ln(r/r_1)}{\ln(r/r_2)},$$

where T, r - temperature and radius of the current point; T_1, r_1, T_2, r_2 - temperatures and radius of the inner and outer surfaces of the cylinder, $T_1 = 100\deg$, $T_2 = 0$, $r_1 = 2\,m$, $r_2 = 3\,m$. Crack length $l_{cr} = 0{,}5\,m$.

Discrete models with allowance for symmetry are designed for a half of cylinder (Fig. 7).

As in the previous test problem, the finite elements in the vicinity of the crack tip are the squares with side l_{FE} corresponding to the ratio $l_{FE}\,/\,l_{cr}=10$ (Fig. 7, *a*) and $l_{FE}\,/\,l_{cr}=20$ (Fig. 7, *b*).



a

b

Fig. 7. Cylinder discrete models of finite elements of different sizes

The calculation results are presented in the form of CIF, calculated by direct method − on the basis of displacements, and by energy method − by the values of *G* and J^*.

The value of the CIF $(K_I, MPa\sqrt{m})$, calculated on the basis of *G* by the method of compliance for a mesh with the *FE* size $l_{FE}\,/\,l_{cr}=20$ in the vicinity of crack tip was taken as the reference. As can be seen, the results of the calculation of CIF by all three methods are almost the same (Table 2).

Table 2

l_{FE} / l_{cr}	Direct method		Method of reaction		Method of compliance $K_I(G)$
	$K_I(u)$	δ, %	$K_I(J^*)$	δ, %	
10	248	4.4	235.4	0.9	235.9
20	238.4	0.4	237.4	0.1	237.6

A test problem about rectangular plate with a crack which uniformly cooled down at $\Delta T = 100\,\text{deg}$ was considered next. Boundary condition is tough pinching along the edges (Fig. 8, *a*). Physical and mechanical properties are similar to the previous test problem. Discrete models are designed for half of the plate with allowance for symmetry and shown in Fig. 8, *b* and Fig. 8, *c* at $l_{FE} / l_{cr} = 10$ and $l_{FE} / l_{cr} = 20$ respectively.

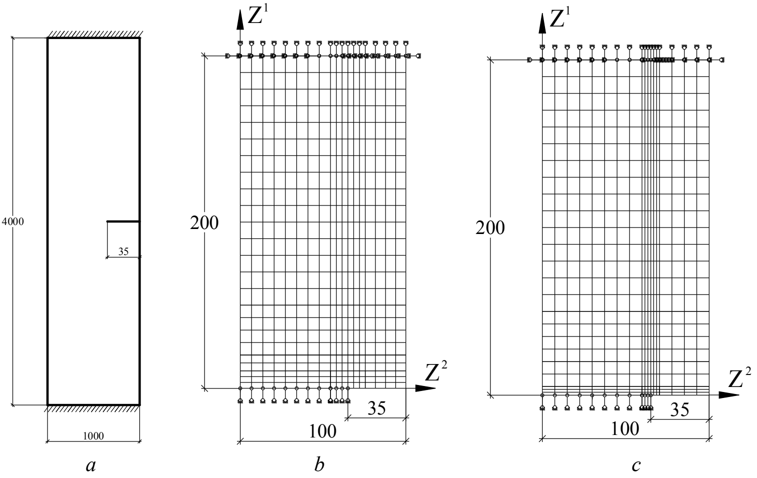


Fig. 8. Rectangular plate with a crack: design sheme (*a*), discrete models of finite elements of different sizes (*b,c*)

The results of the calculation show that, as in the previous problem, the method of compliance and the method of reactions give practically the same result (Table 3, $K_I, MPa\sqrt{m}$). The difference of the results of the direct method is within 5% . The solution of this problem by analytical method and using the ANSYS is given in [6]. The CIF values in ANSYS were calculated by the direct method using the displacement in the node which is closest to the crack tip and by the energy method using the value of the *J*-integral. The obtained results differ from those shown in Table 3 within 3%.

Table 3

l_{FE} / l_{cr}	Direct method		Method of reaction		Method of compliance
	$K_I(u)$	δ, %	$K_I(J^*)$	δ, %	$K_I(G)$
10	519.8	4.8	543.4	0.5	543.1
20	520.6	4.7	546.2	0	546.3

Conclusions. The solution of the test problems showed the reliability of the developed method for determining the parameter J^* in comparison with other methods for bodies with cracks under the action of the volumetric forces of various nature. The results are in good agreement with the data received by other authors in the named publications.

Thus, the approach based on the method of reaction to the calculation of the parameter J^* allows to estimate independently the cracking strength of bodies with cracks, which can justify the reliability of the results in comparison with other methods.

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МЕТОДИКА ВИЗНАЧЕННЯ ПАРАМЕТРА J^* В ДИСКРЕТНИХ МОДЕЛЯХ МЕТОДУ СКІНЧЕНИХ ЕЛЕМЕНТІВ

На основі методу реакцій розроблена методика обчислення параметру J^* методом зміщення підобластей в дискретних моделях методу скінчених елементів. Проведено розв’язання тестових задач. Отримані результати підтверджують ефективність методики.

Ключові слова: механіка руйнування, напіваналітичний метод скінчених елементів, параметр J^*, метод реакцій.

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METHODOLOGY OF DETERMINING OF PARAMETER *J** IN DISCRETE MODELS OF FINITE ELEMENTS METHOD

Fracture mechanics’ parameters are used when the stress-strain state in the vicinity of the crack tip being evaluated. Energy methods have been most developed in determining of the fracture mechanics parameters to date. A method of reactions has been developed in earlier works of authors that makes it possible to determine the *J*-integral in discrete models of the finite element method. However, the use of the J-integral is possible in the absence of voluminous forces. The case of voluminous forces caused by the action of temperature loads is the exception of it. J^* is used as the fracture parameter in this case. A new method for calculating of the parameter J^* is developed in this paper, based on the reaction method. The implementation of this method involves the displacement of the subregions of integration into one finite element. Approbation of the developed approach was carried out on the test problem of deformation of a rectangular plate with a central crack. The obtained results showed that under the action of both surface and voluminous forces on the plate, the values of the parameter J^* coincide with the standard solution. Further, the developed approach was implemented for spatial problems, using discrete models of a semi-analytic finite element method. The determination of J^* is performed in this case at separate points of the crack front for each of which separate volumes of integration and corresponding subregions are stand out. Approbation of the technique in the spatial bodies was carried out on the test problem of the bending of a prismatic body with a lateral notch. Then the developed method was tested on two test problems in the presence of volume forces caused by the action of temperature loads. The results coincided with the values of J^* obtained by other authors with the help of the finite element software ANSYS. Thus, the method of determining of the parameter J^*, developed on the basis of the reaction method, makes it possible to effectively evaluate the stress-strain state in bodies with cracks under the action of surface and voluminous forces.

Keywords: fracture mechanic, semi-analytical finite element method, J^* parametr, reaction method.

Баженов В.А., Пискунов С.О., Шкрыль А.А.

МЕТОДИКА ОПРЕДЕЛЕНИЯ ПАРАМЕТРА J^* В ДИСКРЕТНЫХ МОДЕЛЯХ МЕТОДА КОНЕЧНЫХ ЭЛЕМЕНТОВ

На основе метода реакций разработана методика определения параметра J^* методом смещения подобластей в дискретных моделях метода конечных элементов. Проведено решение тестовых задач. Полученные результаты подтверждают эффективность методики.

Ключевые слова: механика разрушения, полуаналитический метод конечных элементов, параметр J^*, метод реакций.

Баженов В.А., Пискунов С.О., Шкриль А.А. **Методика визначення параметра *J** в дискретних моделях метода скінченних елементів** // Опір матеріалів і теорія споруд. – 2017. – Вип. 99. – С. 33 – 44.

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