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INFLUENCE OF SYSTEM STIFFNESS PARAMETERS AT CONTACT SOFTNESS IN VIBROIMPACT SYSTEM

The possibility of contact character changing by system parameters changing is researching. It is investigated the contact between two bodies in two-degree-of-freedom vibroimpact system. We show which parameters changing can transform the rigid impact in system into the soft one. In the main these parameters are the Young's modulus and geometrical characteristics of the contact zone for both bodies.

Key words: vibroimpact system, impact duration, rigid impact, soft impact, stiffness, Hertz theory, optimization.

Introduction

Vibroimpact machines and equipment are often encountered in many applications of engineering practice. In general systems in which impacts of matching elements occur play an important role in the theory of vibration of mechanical systems. Therefore the study of their dynamic behaviour and vibroimpact movement characteristics in different work condition has got the special interest. Such investigations are developed extensively during the last decades [1-3].

S.G. Kryzhevich writes for ground the actuality of his Doctor's dissertation theme [4]: "Несмотря на потребность прикладной механики и техники в развитии методов исследования инвариантных множеств сильно нелинейных динамических систем, устойчивости их решений и механизмов возникновения хаотических колебаний, а также в исследованиях по анализу конкретных виброударных систем, работы в этих направлениях на настоящий момент еще находятся в начальной стадии" ("The applied mechanics and technique need in development of methods for investigations of invariant strongly nonlinear dynamic systems and their solution stability. The mechanism of chaotic vibration rise and analysis of specific vibroimpact systems are required too. In spite of such necessity the works after these directions are at initial stage at present").

There is the viibroimpact systems (VIS) classifications by different aspects [5]. One of them is impact type characteristic – rigid or soft. Some principal difference between rigid and soft contact were formulated in [6]. The main sign is its duration. Just impact duration dictates the way of its simulation. If impact duration is large impact isn't instantaneous. Its simulation by boundary conditions with Newton's restitution coefficient using based on stereomechanic theory isn't possible [7]. The stiffness of VIS elements causes the impact softness.

The soft impacts take place in engineering very often. The authors write in [8]: "Soft impacts occur in many practical mechanical systems where there is some "cushioning" at the impacting surfaces – meant for reducing the noise and chatter. It can be visualized as a mass impacting not with a hard wall, but with a spring-damper support in front of a wall. The existence of the spring-damper type cushion introduces some special features in the system dynamics".

The investigation of systems which constituted by softly impacting beams and rods of non-negligible mass is fulfilled in [9]. The impact is simulated by force which depends linearly from bodies penetrations one into another. Some virtual spring imitates this force. The values of bodies' displacements and penetrations characterize the impact softness. The authors investigate the influence of virtual spring stiffness at system dynamic behaviour and vibrative motion character.

In our works we studied two models of VIS. The first model is VIS with rigid impact. Its calculation scheme is corresponding to the system with percussive or nonpercussive dynamic damper [10] (Fig. 1,*a*). Another model is VIS with soft impact. Its calculation scheme is corresponding to vibroimpact platform, which is widely used in building industry for concrete mix compaction and concrete products forming [7] (Fig. 1,*b*).

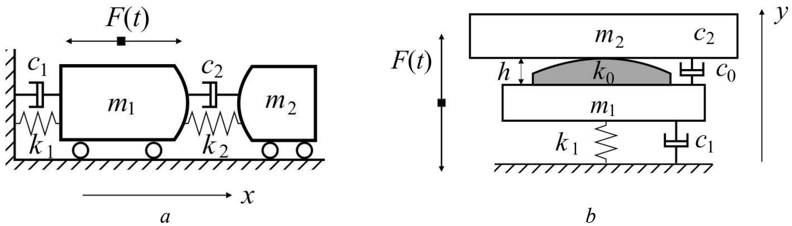


Fig. 1. Calculation schemes of VIS: *a* – with rigid impact; *b* – with soft impact

Contact impact forces in these systems in dependence from time have got the forms that is shown at Fig. 2,*a,b,c*. Numerical parameters values have been written in [7, 10]. The rigid impact is almost instantaneous because its duration is very small. Therefore the contact force graphic in every impact has the form of "stick". On the contrary the duration of soft impact is large; the contact force graphic in every impact has the form of "bell" [11]. It should be noted that the frequency of external loading at vibroimpact platform is larger to 20 times then the frequency of external loading at vibroimpact system with rigid impact. Therefore the scale of time axis at Fig. 2,*a,b* is different for these two models. Fig. 2,*b* shows “stick” in the same time scale which is used for “bells” at Fig. 2,*c* in order the comparison of contact force character to be correct, pure.

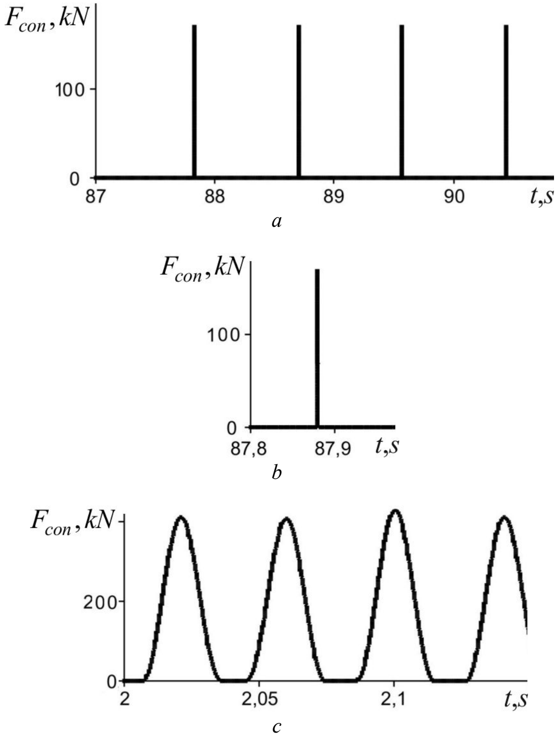


Fig. 2. Character of contact force which gets exited in VIS:
a,b – with rigid impact, *c* – with soft impact

The bodies displacements in independence from time are shown at Fig. 3,*a,b*.

Let us look attentively at the Fig. 3. For the system with rigid impact it is seen well as attached body (thin curve) jumps away from the main one (thick curve) in a moment. It doesn’t penetrate into the main body. On the contrary in the system with soft impact the upper body when falling and shocking the lower one causes its considerable deformation and penetrates into it.

Is it possible to select the parameters of VIS with rigid impact like that the impact in it will become the soft one? This article searches the answer at this question. The parameters influence at the impact duration was also studied in [12].

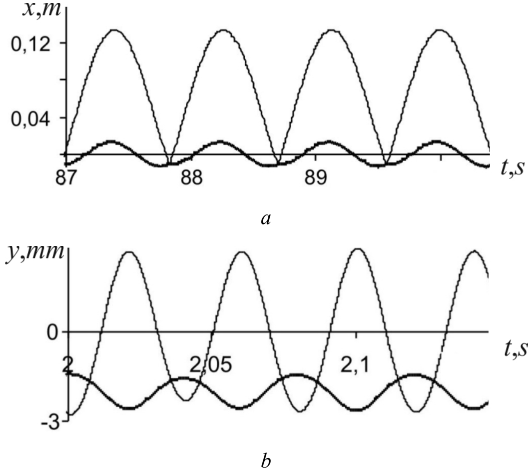


Fig. 3. Bodies displacements in independence from time in VIS:
a – with rigid impact, *b* – with soft impact

1. The main equations

We'll study the problem about impact softening using the scheme of two-of-degree-freedom VIS of two bodies which is shown at Fig. 1,*a*. This VIS is described in [11]. Let us remind of it shortly. The system is formed by the main body and attached one. Bodies are connected by linear elastic springs and dampers. The main body is subjected to the action of periodical external force:

$$F(t)=\lambda F_0 \cos(\omega t+\varphi_0)\,, \tag{1}$$

where λ is parameter of its intensity. We consider the elastic collinear impacts with low velocities. The contact surfaces are smooth, curvilinear, without roughness. For impact simulation we use the Hertz's contact interaction force based on quasistatic Hertz's theory [13]:

$$F_{con}(z)=K[H(z)z(t)]^{\frac{3}{2}}, \tag{2}$$

$$K=\frac{4}{3}\frac{q}{(\delta_1+\delta_2)\sqrt{A+B}},\quad \delta_1=\frac{1-\mu_1^2}{E_1\pi},\quad \delta_2=\frac{1-\mu_2^2}{E_2\pi},$$

where $z(t)$ is the relative closing in of bodies due the local deformation in contact zone, $z(t)=x_1-x_2$, $H(z)$ is the Heaviside step function:

$$H(z)=\begin{cases}1, & z\ge 0\\ 0, & z<0\end{cases},$$

μ_i and E_i are respectively Poisson's ratios and Young's modulus for both bodies, *A, B* and *q* are the characteristics of contact zone local geometry. We consider that these surfaces are spherical, then $A=B=1/2R_1+1/2R_2$, where R_1, R_2 are the contact surfaces radiuses. The Hertz/s theory takes into account only local deformations in contact zone.

In [14] the authors examine the limitations for using of Hertz's theory in different individual cases. For example the contact surfaces aren't Herzian, impact velocities are large and the plastic deformations occur, there is conformal contact between bodies, the impacts aren't pure elastic and energy dissipation has to take into account. The authors write: "The Hertzian contact theory remains the foundation for almost all of the available force models, but by itself, it is not appropriate for most impacts in practice, due to the amount of energy dissipated during the impact". In [14] there are the set of contact forces models which enlarge and define more precisely the Hertz's theory. They complete the expression for contact force by additional terms that take into account damping. The authors write in conclusions: "There is no doubt that the biggest landmark in contact mechanics was the work of Hertz for static elastic contacts. The Hertzian contact approach is based on the theory of elasticity and still remains the foundation for elastic and dissipative contact force models available in the literature". The using of less rough impact models for VIS (for example the wave theory) causes considerable difficulties due to repeated impacts. Therefore Hertz's theory is widely used for analysis of VIS dynamics now too. The words of Prof. Ivanov A.P. are proper here: "Выбор той или иной модели удара для решения конкретной задачи связан с компромиссом между простотой и реалистичностью, достигнуть которого на практике, однако, удается редко" ("The choice of one or another impact model for specific problem solution is connected with compromise between simplicity and reality. You can rarely to reach such compromise in practice") [3].

Movement equations of the system have the form:

$$\ddot{x}_1=-2\xi_1\omega_1\dot{x}_1-\omega_1^2x_1-2\xi_2\omega_2\chi(\dot{x}_1-\dot{x}_2)-\omega_2^2\chi(x_1-x_2+D)+$$
$$+\frac{1}{m_1}[F(t)-F_{con}(x_1-x_2)],$$
$$\ddot{x}_2=-2\xi_2\omega_2(\dot{x}_2-\dot{x}_1)-\omega_2^2(x_2-x_1-D)+\frac{1}{m_2}F_{con}(x_1-x_2), \qquad (3)$$

where $\omega_1=\sqrt{k_1/m_1}$, $\omega_2=\sqrt{k_2/m_2}$; $\xi_1=\dfrac{c_1}{2m_1\omega_1}$, $\xi_2=\dfrac{c_2}{2m_2\omega_2}$; $\chi=\dfrac{m_1}{m_2}$, ω_1,ω_2 - partial vibration frequency, $F_{con}(x_1-x_2)$ – contact interaction force, it simulates the impact and works only during the impact. Initial conditions are:

$$x_1(0)=0, x_2(0)=D, \dot{x}_1(0)=0, \dot{x}_2(0)=0.$$

Numerical parameters of VIS are given in [10] and in Table 1 further.

2. Impact softening by system parameters changing

The clear criterion of impact rigidness or softness is absent. The typical trait of impact softness is its duration. Is it instantaneous or not? Let us examine the value – the coefficient of the relative impact duration $k_{con} = T/T_{con}$, where T – period of external loading (1), $T = 2\pi \cdot \omega^{-1}$, T_{con} – the time of impact i.e. the time of contact between bodies.

The impact duration $T_{con} = 7,81 \cdot 10^{-4} s$, the coefficient of the relative impact duration $k_{con} = T/T_{con}$ =1113,3 for the motion with rigid impact that is shown at Fig. 2,*a*, Fig. 3,*a*. Here VIS has parameters that are in Table 1.

Table 1

Bodies' characteristics	**Main body**	**Attached body**
Mass m_i, kg	1000,0	100,0
Partial vibration frequency $\omega_i, rad \cdot s^{-1}$	6,283	5,646
Young's modulus $E_i, H \cdot m^{-2}$	$2,099 \cdot 10^{11}$	$2,099 \cdot 10^{11}$
Contact surface radius R_i, m	2	2
Coefficients A, B, m^{-1}, q	A=0,5 B=0,5 q=0,319	
External loading amplitude F_0, H		500,0
External loading frequency $\omega, rad \cdot s^{-1}$		7,23

Let us examine what way can be chosen for decrease the coefficient of the relative impact duration k_{con} (i.e. increase the impact duration T_{con}) by changing the parameters of VIS, in the main the stiffness parameters. For this problem decision we use numerical effective method based on theory and techniques of nonlinear programming for solution the problem of parameters optimization. We use the modified steepest descent method – gradient projection method with correction of residual with constraints [10,15,16]. We formulate the minimax problem like that: to find such parameters of VIS which will provide the smallest value of relative impact duration coefficient k_{con}, i.e. the largest value of impact duration T_{con}. We seek for the objective function $k_{con} = T/T_{con}$ by deciding the problem about steady-state exiting vibrations of VIS under the concrete system parameters. Numerical integration of motion equations (3) was fulfilled by Runge-Kutta 4th order method. The calculation of objective function gradient takes into account the constraints laid on the parameters. The obtained value determines the next optimization step. We investigated the influence of such parameters at impact duration: the attached body's mass m_2; the joining spring's stiffness k_2 (it comes into the partial frequency expression (3)); Young's modulus of both system bodies E_1, E_2; the radiuses of contact surfaces for both bodies R_1, R_2 in assumption that these surfaces are spherical (these radiuses comes into expressions of coefficients A and B in formula for Hertz's force (2)). The other parameters weren't changed. We have obtained such results. The impact duration was enlarged considerably under influence of these parameters changing. The graphics of displacements and contact force are shown at Fig. 4 for parameters values which are given in Table 2.

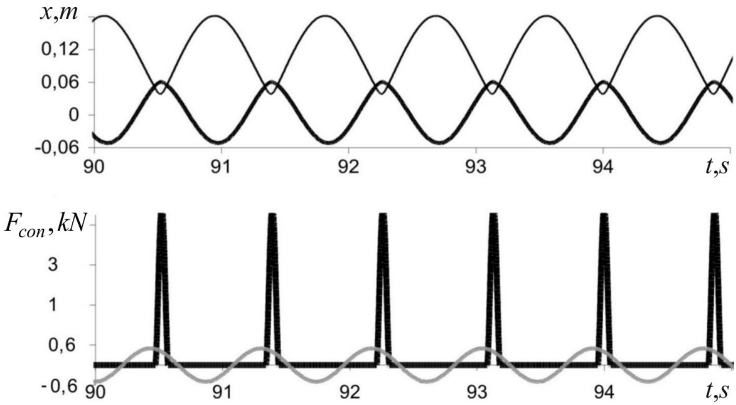


Fig. 4. Characters of (1,1)-periodical vibration regime ($k_{con} = 8,868$)

Table 2

Bodies' characteristics	Main body	Attached body
Mass m_i, kg	1000,0	310,0
Partial vibration frequency $\omega_i, rad \cdot s^{-1}$	6,283	3,606
Young's modulus $E_i, H \cdot m^{-2}$	$1{,}4 \cdot 10^{10}$	$1{,}0 \cdot 10^{6}$
Contact surface radius R_i, m	2	2
Impact duration T_{con}, s		0,098
Coefficient of impact duration k_{con}		8,868

At the displacement graphic (Fig.4) we see how attached body (thin curve) penetrates into the main one (thick curve) due their local deformation. The contact force graphic (black curve) can be compared with the external loading graphic (grey curve). It isn't similar at "stick" already, it draws near the "bell". The same picture is observed under the other parameters values obtained after VIS optimization (Table 3, Fig. 5).

Table 3

Bodies' characteristics	Main body	Attached body
Mass m_i , kg	1000,0	231,0
Partial vibration frequency $\omega_i, rad \cdot s^{-1}$	6,283	3,606
Young's modulus $E_i, H \cdot m^{-2}$	$1{,}4 \cdot 10^{10}$	$2{,}44 \cdot 10^{5}$
Contact surface radius R_i, m	1	0,5
Impact duration T_{con}, s		0,164
Coefficient of impact duration k_{con}		5,301

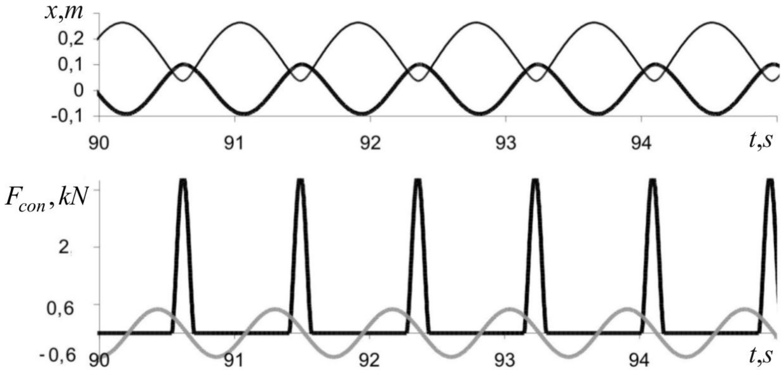


Fig. 5. Characters of (1,1)-periodical vibration regime ($k_{con} = 5,301$)

The best result (i.e. the biggest value of impact duration) for *T*-periodic one-blow vibration regime was obtained for such VIS parameters values (Table 4, Fig. 6).

Table 4

Bodies' characteristics	Main body	Attached body
Mass m_i , kg	1000,0	310,0
Partial vibration frequency $\omega_i, rad \cdot s^{-1}$	6,283	3,606
Young's modulus $E_i, H \cdot m^{-2}$	$2{,}44 \cdot 10^{5}$	$2{,}1 \cdot 10^{11}$
Contact surface radius R_i, m	0,1	0,09
Impact duration T_{con}, s		0,233
Coefficient of impact duration k_{con}		3,732

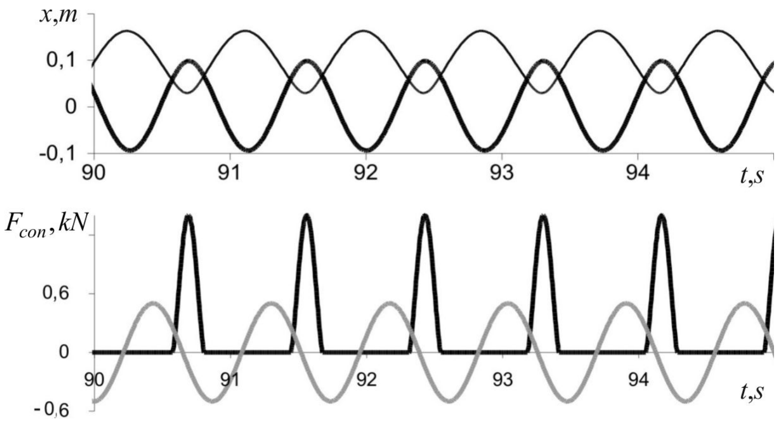


Fig. 6. Characters of (1,1)-periodical vibration regime ($k_{con} = 3,732$)

Let us notice some unexpected result obtained during researches: decrease of joining spring stiffness doesn't provide the considerable impact softening. The contact forces changing during one impact at different values of joining spring stiffness are shown at Fig. 7.

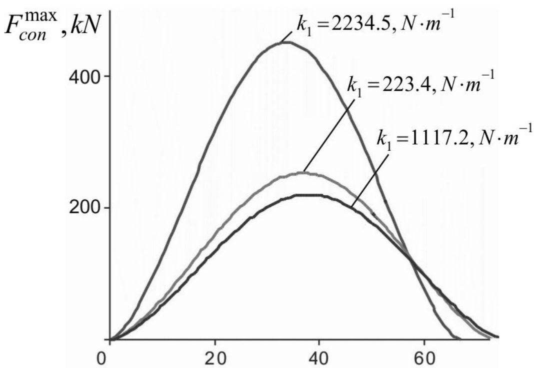


Fig. 7. Contact force dependence from joining spring stiffness

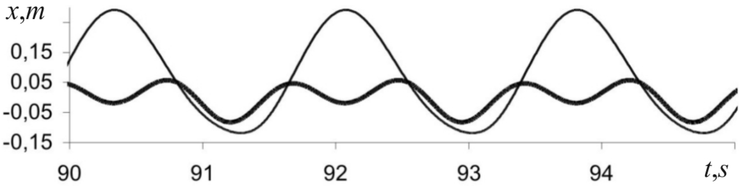
Decrease of stiffness twice and ten times diminishes the contact impact force almost equally and practically doesn't increase the impact duration.

Further decrease of stiffness causes the loss of stability for *T*-periodic one-blow regime and rise of *nT*-periodic multi-blow regimes. Let us notice by the way that phenomenon of existence of periodical regimes with large (in the limiting case infinity) number of impacts per period has got the name "chatter" ("rattle") or infinity impact regime (in Russian literature "стук"). It is one of the significant reasons of appearance strange attractor i.e. rise of the chaotic vibrations [4].

For further impact softening we can increase the impact duration some more when decreasing bodies' materials Young's modulus and changing the contact surfaces radiuses. But then the motion character changes too – motion becomes nT-periodic. Such regime is shown for example (Table 5, Fig. 8).

Table 5

Bodies' characteristics	**Main body**	**Attached body**
Mass m_i , kg	1000,0	310,0
Partial vibration frequency ω_i, $\text{rad}\cdot\text{s}^{-1}$	6,283	3,606
Young's modulus E_i, $\text{H}\cdot\text{m}^{-2}$	$2{,}0\cdot10^4$	$8\cdot10^{10}$
Contact surface radius R_i, m	1,5	0,9
Impact duration T_{con}, s	0,837	
Coefficient of impact duration k_{con}	1,039	



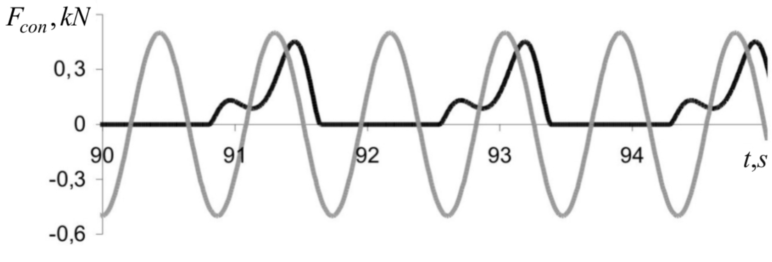


Fig. 8. Characters of (2,1)-periodical vibration regime ($k_{con} = 1{,}039$)

The impact is very soft. The penetration of the attached body into the main one is significant. The contact force graphic is pronounced "bell". The motion is $2T$-periodic.

The impact duration is increasing but the contact force is decreasing when the impact is becoming more soft. Fig. 9 shows this phenomenon for third values of relative impact duration coefficient very clearly.

Let us pay attention that the using of nonlinear contact Hertz's force provides the possibility of simulation both rigid and soft impact, the possibility of finding the contact force value and its change law in time, the possibility of impact duration calculation. The using of the boundary conditions with Newton's restitution coefficient based at stereomechanic theory doesn't provide such possibility [17, 18]. The using of Hertz's force provides also to take into account more diversified influence of system parameters then the using of force which depends linearly from bodies' rapprochements and has got only one proportional coefficient characterized the stiffness.

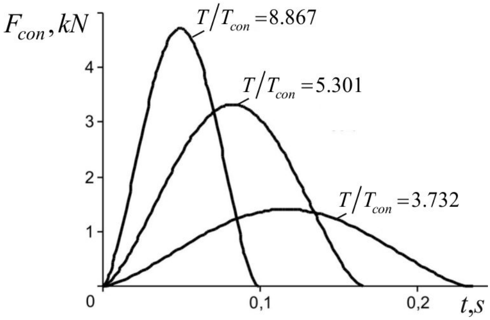


Fig. 9. Contact force during one period of impact in dependence from its duration

Conclusions

The investigations fulfilled in this work allow to make the next conclusions.

1. Parameters of VIS with rigid impact may be changed like that the impact will become the soft one. Gradient projection method with correction of residual with constraints gives the possibility to find the optimal system parameters which will provide the maximum impact duration.

2. The using of nonlinear contact force based on quasistatic Hertz's theory allows simulating the impact in VIS with both rigid and soft impact. Its using gives the possibility to find not only maximum value of contact force but its changing in time too, provides the calculation of impact duration. The using of this force provides also the diversified taking into account of the system parameters influence at its dynamic behaviour.

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ТВЕРДИЙ ТА М‘ЯКИЙ УДАР В ВІБРОУДАРНИХ СИСТЕМАХ

Досліджується можливість зміни характеру контакту між тілами двохмасової віброударної системи з двома ступнями вільності шляхом зміни її параметрів. Показано, яка зміна параметрів може перетворити твердий удар в системі на м‘який. В основному, ці параметри такі: модулі пружності Юнга та геометричні характеристики зони контакту для обох тіл.

Ключові слова: віброударна система, тривалість удару, твердий удар, м‘який удар, жорсткість, теорія Герца, оптимізація.

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ТВЕРДЫЙ И МЯГКИЙ УДАР В ВИБРОУДАРНЫХ СИСТЕМАХ

Исследуется возможность смены характера контакта между телами в двухмассовой виброударной системе с двумя степенями свободы путем изменения её параметров. Показано, какое изменение параметров может преобразовать твердый удар в системе в мягкий. В основном, эти параметры таковы: модули упругости Юнга и геометрические характеристики зоны контакта для обоих тел.

Ключевые слова: виброударная система, длительность удара, твердый удар, мягкий удар, жесткость, теория Герца, оптимизация.