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INVARIANT TORUS BREAK-DOWN IN VIBROIMPACT SYSTEM – ROUTE TO *CRISIS* ?

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Abstract. Chaotic vibrations of dynamical systems and their routes to chaos are interesting and investigated subjects in nonlinear dynamics. Particularly the routes to chaos in non-smooth dynamical systems are of the special scientists' interest. In this paper we study quasiperiodic route to chaos in nonlinear non-smooth discontinuous 2-DOF vibroimpact system. The break-down of invariant torus or of the closed curve occurs just under the quasiperiodic route to chaos. Is it route to crisis? In narrow frequency range different oscillatory regimes have succeeded each other many times under very small control parameter varying. There were periodic subharmonic regimes − chatters, quasiperiodic, and chaotic regimes, the zones of prechaotic and postchaotic motion. The hysteresis effects (jump phenomena) occurred for increasing and decreasing frequencies. The observed chaos was the transient one.The chaoticity of obtained regime has been confirmed by typical views of Poincaré map and Fourier spectrum, by the positive value of the largest Lyapunov exponent, and by the fractal structure of Poincaré map. These investigations confirm the theory by Newhouse, Ruelle, and Takens who suggested a new bifurcation scenario where a periodic solution produces subsequently a torus and then a strange attractor.
Keywords: vibroimpact system, dynamical behaviour, quasiperiodic, chaotic, subharmonics, Poincaré map, Fourier spectrum, Lyapunov exponent, fractal structure.

1. Introduction

Nonlinear dynamics is relatively young science. It has begun to develop rapidly only at the end of 20-*th* century. One of the most interesting and explored subjects on nonlinear dynamics are the chaotic vibrations. The routes of dynamical systems to chaos are of the special scientists' interest. There are many papers, monographs and textbooks about dynamic behaviour in general and routes to chaos in particular in nonlinear systems [1-6].

Just deterministic chaos is not the exceptional regime of dynamical system behaviour. On the contrary such regimes are observed in many dynamical systems in mathematics, physics, mechanics, biology, medicine. Recent such investigations appear in economics and sociology more and more often. For example Professor D. Volchenkov (Texas Tech University) is doctor of sciencies, expert on theoretical physics, mathematics, and nonlinear dynamics. He is editor in chief of International Journal "Discontinuity, Nonlinearity, and Complexity". Nevertheless at present his studies devote to sociology problems [7, 8].

Therefore the investigations on nonlinear dynamics in general and on chaotic dynamics in particular are one of the arterial ways in the contemporary natural science development.

The term *crisis* was one of the new words coined by C. Grebodgi et al. [9]. It is used to denote a sudden change in the chaotic state when some system parameter is changed. For example a system initially in a chaotic state may suddenly become periodic. Or chaotic motion which was originally confirmed to a limited range of $x(t)$ may suddenly expand to a broad range $x(t)$.

It is known that the studying of non-smooth dynamical systems with discontinuous right-hand side has some difficulties. So dynamical processes in non-smooth systems are studied less. In works [10,11] authors divide non-smooth discontinuous dynamical systems into three types according to their degree of discontinuity. There are among them Fillipov systems and the impacting systems with velocity reversals. Moreover the systems with impacts between its elements have the grossest form of nonlinearity and the non-smoothness. Many new phenomena unique to non-smooth systems are observed under variation of system parameters. Recently the investigations of such systems are developed rapidly. Especially systems with impacts are of the particular interest for scientists. Exactly in such systems the discontinuous dangerous bifurcations are arising under system parameters variation. Just such hard bifurcations can portend the *crisis*.

Vibroimpact system is strongly nonlinear non-smooth one; the set of differential equations of motion contains the discontinuous right-hand side. The studying of vibroimpact system dynamic behaviour both in general and for concrete system is of the special interest. In particular the routes to chaos in such systems also are of the special interest. It is well known that completely deterministic dynamic system may begin to behave by unforeseen chaotic manner when any accidental influence is absent. However, in this unpredictability it is possible to identify a number of regularities in the system behaviour which distinguishes this phenomenon from the classical random processes. Moreover, in contrast to the classical random processes, the phenomenon of deterministic chaos can be reproduced in natural, laboratory and numerical experiments. It is known three main routes to chaos in dynamical systems [1,3]:

- period-doubling route to chaos – the most celebrated scenario for chaotic vibrations;
- quasiperiodic route to chaos;
- route to chaos via intermittency by Pomeau and Manneville: the long periods of periodic motion with bursts of chaos; as one varies a parameter the chaotic bursts become more frequent and longer [12].

The invariant torus break-down, or breakup of the closed curve provides just quasiperiodic route to chaos. In [13] the authors consider the mathematical side of this problem. They suggest some hypotheses and formulate theorem where three possibilities are given:

- The stable and unstable periodic orbits vanish through a bifurcation.
- Stable and unstable manifolds of the unstable periodic orbit intersect tangentially to form a homoclinic orbit.
- The stable periodic orbit looses stability.

The authors write at the paper end:”The further experimental and numerical studies of mentioned problems have to confirm or to refuse these hypotheses”.

In [4] there is section “Universal properties of the route from quasiperiodisity to chaos”. The author considers this problem in the main via maps. Recently the problem of torus break-down was considered for example in [14-17].

We use different characteristics in order to be sure that obtained oscillatory regime is chaotic one.

Poincaré maps are one of the principal ways of recognizing chaotic vibrations in low-degree of freedom problems. Poincaré maps and phase plane portraits can often provide graphic evidence for chaotic behaviour and the fractal properties of strange attractors. Poincaré maps help to distinguish between various qualitative states of motion such as periodic, quasiperiodic, or chaotic. But quantitative measures of chaotic dynamics are also important and in many cases are the only hard evidences for chaos. One of the significant characteristics is Fourier distribution of frequency spectra. The difference between chaotic and quasiperiodic motion can be detected by taking the Fourier spectrum of the signal. A quasiperiodic motion will have the well-pronounced peaks at basic frequencies and at their combinations, chaotic motion – a broad continuous spectrum of Fourier components.

Chaos in dynamics implies a sensitivity of the outcome of a dynamical process to changes in initial conditions. Small uncertainties in initial conditions lead to divergent orbits in the phase space. Small changes in initial conditions (or in some other parameters such as, for example, the amplitude or frequency of exciting force, damping coefficient) can dramatically change the type of output from a dynamical system.

Lyapunov exponents characterize the kind of dynamical system motion because they measure the divergence rate of nearby trajectories. In order to have a criterion for chaos one need only calculate the largest exponent λ which tells whether nearby trajectories diverge ($\lambda > 0$) or converge ($\lambda < 0$) on the average. Its sign is chaos criterion. For regular motions $\lambda \le 0$, but for chaotic motion $\lambda > 0$ that is positive Lyapunov exponent imply chaotic dynamics.

There are some difficulties with Lyapunov exponents determination for non-smooth systems especially for discontinuous systems with impacts. These difficulties are caused by the discontinuity of motion equations right-hand sides. The Jacobian matrix which is used in Benettin’s algorithm of Lyapunov exponent calculation is also discontinuous. At present there are several propositions for Lyapunov exponents calculation in non-smooth systems. The authors of these propositions describe their own methods for such estimation [18-20].

One can consider the fractal structure of Poincaré map as visit card of chaotic motion. When the motion is chaotic, a mazelike, multisheeted structure in section may appear. This threadlike collection of points seems to have further structure when examined on a finer scale. The term *fractal* characterizes such Poincaré patterns. So the fractal dimension of chaotic attractor is one of the principal measures of chaos.

In [1] the author advises not to rely on one measure of chaos in dynamical experiments, but to use two or more techniques such as Poincaré maps, Fourier spectra, Lyapunov exponents or fractal dimension measurements before pronouncing a system chaotic or strange.

When we say about quasiperiodic route to chaos we have to bear in mind that in this case the whole picture is found sufficiently complicated. Its many aspects remain not studied to the end so far. Attractor evolution under governing parameter changing may be various and complicated. Quasiperiodic and periodic regimes may alternate and undergo different bifurcations.

The goals of this paper are the following:

1) To study by numerical simulation the picture of invariant torus (or closed curve) break-down that is the quasiperiodic route to chaos in 2-DOF two-body vibroimpact system.

2) To evaluate the chaoticity of obtained oscillatory regimes by several quantitative and qualitative characteristics such as Poincaré maps, Fourier spectra, the largest Lyapunov exponents, and fractal structure of Poincaré map.

3) To confirm or to refuse the hypotheses about breakup of invariant torus formulated earlier.

2. The background for studying of quasiperiodic route to chaos in vibroimpact system

We have studied the dynamic behaviour of 2-DOF two-body vibroimpact system (Fig. 1) in our previous works [21-23]. Therefore we'll give only short problem description.

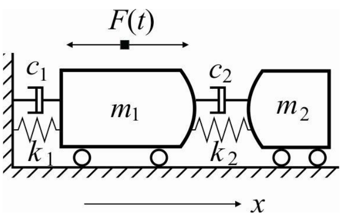


Fig. 1

This model is formed by the main body m_1 and attached one m_2, which can play the role of percussive or non-percussive dynamic damper. Bodies are connected by linear elastic springs with stiffness k_1 and k_2 and dampers with damping coefficients c_1 and c_2. (The damping force is taken as proportional to first degree of velocity with coefficients c_1 and c_2.)

The differential equations of its movement are:

$$\ddot{x}_1=-2\xi_1\omega_1\dot{x}_1-\omega_1^2x_1-2\xi_2\omega_2\chi(\dot{x}_1-\dot{x}_2)-\omega_2^2\chi(x_1-x_2+D)+\frac{1}{m_1}[F(t)-F_{con}(x_1-x_2)],$$

$$\ddot{x}_2=-2\xi_2\omega_3(\dot{x}_2-\dot{x}_2)-\omega_1^2(x_2-x_1-D)+\frac{1}{m_1}F_{con}(x_1-x_2), \qquad (1)$$

where $\omega_1=\sqrt{k_1/m_1}$, $\omega_2=\sqrt{k_2/m_2}$; $\xi_1=\dfrac{c_1}{2m_1\omega_1}$, $\xi_2=\dfrac{c_2}{2m_2\omega_2}$; $\chi=\dfrac{m_2}{m_1}$.

External loading is periodic one: $F(t)=P\cos(\omega t+\varphi_0)$, $T=2\pi/\omega$ is its period.

Impact is simulated by contact interaction force F_{con} according to contact quasistatic Hertz's law:

$$F_{con}(z)=K[H(z)z(t)]^{3/2},$$

$$K=\frac{4}{3}\frac{q}{(\delta_1+\delta_2)\sqrt{A+B}}, \qquad (2)$$

$$\delta_1=\frac{1-\mu_1^2}{E_1\pi}, \quad \delta_2=\frac{1-\mu_2^2}{E_2\pi},$$

where $z(t)$ is the relative closing in of bodies, $z(t)=x_2-x_1$, A, B, and q are constants characterizing the local geometry of the contact zone; μ_i and E_i are respectively Poisson's ratios and Young's modulus for both bodies, $H(z)$ is the discontinuous step Heviside function. The numerical parameters of this system are following:

m_1 =1000 kg, ω_1=6.283 rad/s, ξ_1= 0.036, E_1= 2.1·1011 N·m^2, μ_1=0.3,
m_2 = 100 kg, ω_2=5.646 rad/s, ξ_2= 0.036, E_2= 2.1·1011 N·m^2, μ_2=0.3,
P=500 N, A=B=0.5 m^{-1}, q=0.318.

We have obtained loading curves [22] and amplitude-frequency response [23] in wide range of control parameter by parameter continuation method. Periodic motion stability or instability was determined by matrix monodromy eigenvalues that is by Floquet multipliers' values. The periodical solution is becoming unstable one if even though one Floquet multiplier leaves the unit circle in complex plane that is its modulus becoming more than unit.

Global view of amplitude-frequency response for both vibroimpact system bodies is presented at Fig. 2 in wide range of excitation frequency. The upper curve corresponds to attached body (m_2), the lower one – to main body (m_1). Unstable regimes are dotted by grey colour. At axis of ordinates we have semi-amplitude $A_{\max}$. It means half the peak-to-peak amplitude. For the nonharmonic oscillation it is calculated by the formula $A_{\max}=\dfrac{|x_{\max}|+|x_{\min}|}{2}$.

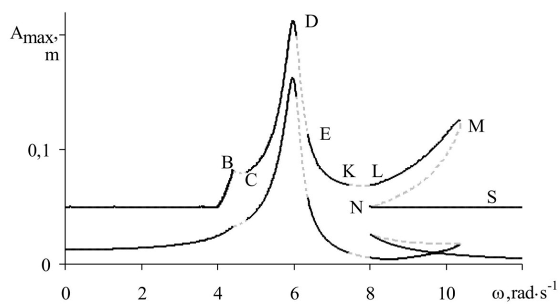
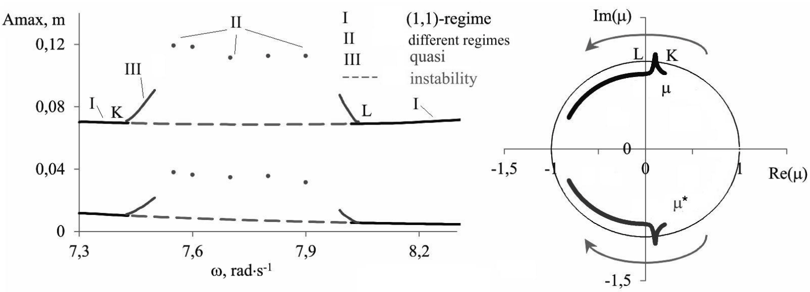


Fig. 2. Amplitude-frequency response

There are some regions of instability of main (1,1)-regime[1] (T-periodic regime with 1 impact per cycle): BC, DE, KL, MN. In this paper we study dynamic behaviour of vibroimpact system in frequency range 7.45 rad/s<ω<8.0 rad/s that is at region KL. Partial view of amplitude-frequency response in instability zone KL is presented at Fig. 3(a). At points K and L stable (1,1)-regime is losing stability, the quasiperiodic regimes are arising as a result of Neimark-Sacker bifurcations. The two complex conjugate multipliers μ and μ* are leaving the unit circle (Fig. 3(b)).

[1] The mark (n,k) means nT-periodic vibration with k impacts per cycle [24], T is period of external loading $T=2\pi/\omega$.



(a) (b)

Fig. 3. Partial view of amplitude-frequency response (a) and multipliers behaviour (b) in *KL* region

After the stability loss at bifurcation points *K* and *L* the second basic oscillatory frequency is arising $\omega_1 = \frac{1}{T}(\arg\ \mu + 2k\pi)$, $k = 0, \pm 1, \pm 2, \ldots$ (the argument of complex number is determined with accuracy $\pm 2k\pi$) [3]. This frequency ω_1 is not commensurate with first basic frequency ω. So the branching dynamical state is quasiperiodic one. Simultaneous time trace of phase plane motion and Poincaré map of this regime for $\omega = 7.46$ rad/s are depicted at Fig. 4. Here and further phase trajectories and Poincaré maps are presented for the main body m_1. Its Fourier spectrum in logarithmic scale is also shown at Fig. 4. We see Poincaré section to be closed curve, and Fourier spectrum has the well-pronounced peaks at two basic frequencies ω and ω_1 and at their combinations, what is typical for quasiperiodic motion.

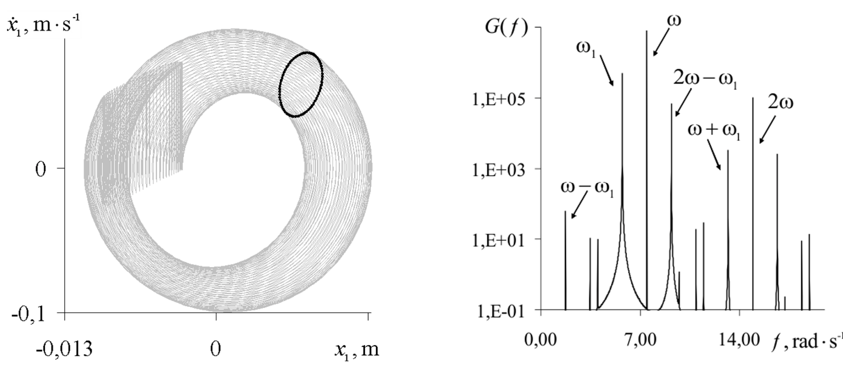


Fig. 4. Poincaré map and Fourier spectrum for quasiperiodic regime

Now we'll look at the system dynamic behaviour between points *K* ($\omega = 7.45$ rad/s) and *L* ($\omega = 8.0$ rad/s) that is after Neimark-Sacker bifurcations.

3. Vibroimpact system dynamic behaviour after Neimark-Sacker bifurcations

We'll see that soon after these bifurcations the quasiperiodic motion will be destroyed. We'll observe the breakdown of closed curves and of invariant torus which are typical for quasiperiodic motion. We'll see new regimes and their alternation – subharmonic periodic vibrations that is regimes with long period and big impact number per cycle ("chatter"). The appearance of subharmonic periodic vibrations is one characteristic precursor to chaotic motion. And the chaotic motion on strange attractor will not keep it waiting as a matter of fact. One of the signs of impending chaotic behaviour in dynamical system is a series of changes in motion nature as control parameter is varied. We'll see some transitional zones of prechaotic or postchaotic state. What do "transitional zones" mean? We cannot call these regimes both periodic or quasiperiodic and chaotic because their characteristics are contradictory ones. So we call they as transitional zones as Prof. F. Moon writes in his famous textbook [1]. We think maybe we'll be able to "catch" intermittency at some frequency? It is future work. Prof. A.Yu. Shvetz (National Technical University of Ukraine "KPI") [6, 25] identifies intermittency with invariant measure helping. We think that it would be nice to fulfill the wavelet analysis of these signals in order to obtain sure quantitative characteristics of intermittency [26, 27].

Now let us have a more detail look at vibroimpact system states which are realizing in this frequency range between Neimark-Sacker bifurcations that is between points K and L. It is known that the sign of largest Lyapunov exponent λ determines sufficiently well the kind of oscillatory motion: the negative sign $\lambda < 0$ corresponds to periodic regimes, the positive sign $\lambda > 0$ − to chaotic ones, and $\lambda \approx 0$ − to quasiperiodic oscillatory regimes. It is known that Lyapunov exponent estimation for non-smooth nonlinear systems has some difficulties because just discontinuity of the right-hand side of motion differential equations. We have written about the largest Lyapunov exponent estimation in non-smooth vibroimpact system in [28]. Now we use the Benettin's algorithm. So far as step Heviside function $H(x_1 - x_2)$ is discontinuous one we must take into attention zero and nonzero for this function under integration both initial equations (1) and equations in variations that are used in Benettin's algorithm. We integrate these equations by the program ode23s (MATLAB® ODE solvers). This program integrates the systems of stiff differential equations.

So the plot of the largest Lyapunov exponent dependence on control parameter shows clearly the whole motion picture at this frequency range (Fig. 5). At this Fig. we see the change of system dynamic states when the control parameter (exciting frequency) is varied. The frequency ranges where the largest Lyapunov exponent is negative ($\lambda < 0$) correspond to periodic regimes, where $\lambda > 0$ − to chaotic ones, and where $\lambda \approx 0$ − to quasiperiodic oscillatory regimes.

Let us now discuss more in details the route to chaos from quasiperiodic regime.

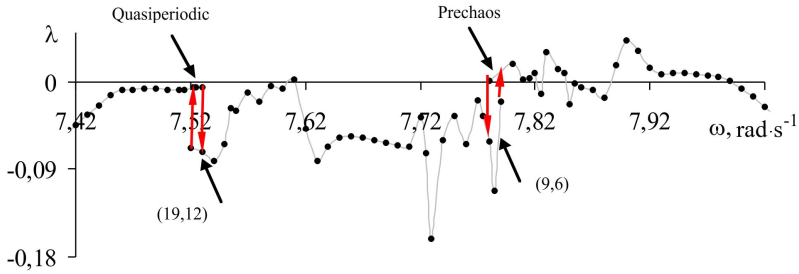


Fig. 5. The largest Lyapunov exponent dependence on control parameter

We observe the hysteresis effect (jump phenomenon)[2] in very narrow frequency range $7.52 \text{ rad/s} < \omega < 7.53 \text{ rad/s}$, where quasiperiodic and (19,12)-periodic regimes are existing. The arising of one or the other regime depends on history that is on initial conditions. It is the region where both periodic and quasiperiodic motions can exist and the precise motion that will result may be unpredictable.

The (19,12)-periodic regime is existing some more under short frequency varying.

Then we see the short zone of transition from (19,12)-periodic regime to other periodic regimes with long periods and big impact numbers per cycle ("chatter" or "rattle"). Transition is beginning at $\omega = 7.55 \text{ rad/s}$ when thirteenth impact is adding to 12 existing ones. At Fig. 6 we clearly see this thirteenth impact. Then there are regimes with very long periods, the Poincaré maps show big points numbers which after all form almost closed curve under $7.59 \text{ rad/s} \le \omega \le 7.61 \text{ rad/s}$. The largest Lyapunov exponent is decreasing tenfold. So this regime looks highly quasiperiodic one if we look at its Poincaré map. But its Fourier spectrum (in logarithmic scale) is board and continious, such as under chaotic motion (Fig. 7,(d)). These characteristics contradicts one another, therefore we think that we cannot call this regime both quasiperiodic and chaotic. It is transitional motion. At Fig. 7 we show phase trajectories and Poincaré maps for (19,12)-periodic regime, chatter, and transitional regime.

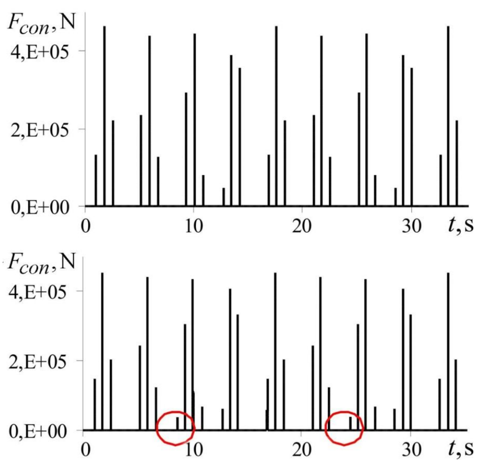


Fig. 6. Contact forces for (19,12)-regime (ω=7.54 rad·s^{-1}) and for (19,13)-regime (ω=7.55 rad·s^{-1})

[2] We consider hysteresis effect as dependence of the system state on its history (the system manifests hysteretic features in the transition between different types of motion) [1].

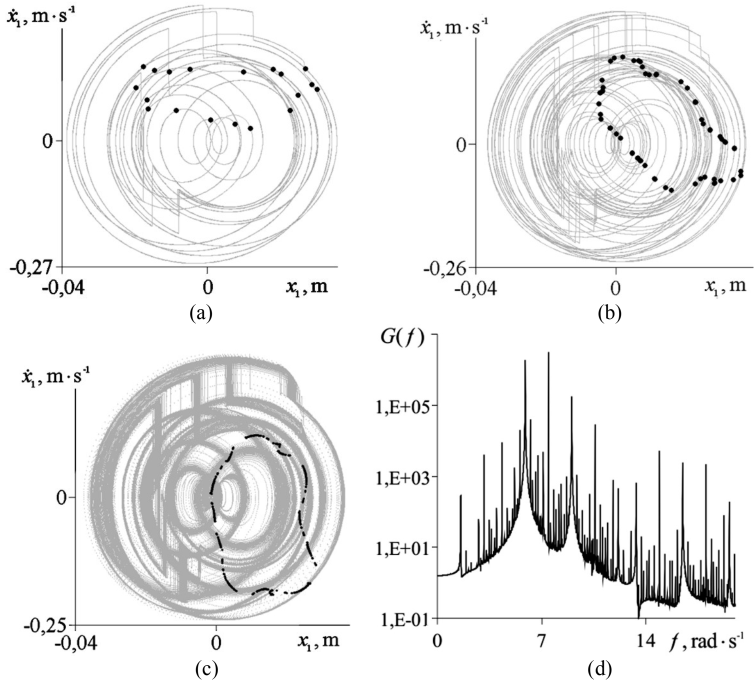


Fig. 7. Phase trajectories and Poincaré maps for: (a) (19,12)-periodic regime $\omega = 7.54$ rad/s , $\lambda = -0.081$; (b) chatter $\omega = 7.58$ rad/s , $\lambda = -0.020$; (c) transitional regime $\omega = 7.61$ rad/s , $\lambda = 0.0027$; (d) Fourier spectrum for transitional regime, $\lambda = 0.0027$

When we are following the Fig. 5 from the left to the right we see the relatively big frequency range where periodic regimes are realizing up to hysteresis phenomenon. There are subharmonics – (14,10) and (23,17)-periodic regimes which sharply replace each other under $\omega = 7.72$ rad/s . Subharmonics play an important role in prechaotic vibrations so far as their appearance in frequency spectrum often is a characteristic precursor to chaotic motion. There may be in fact many patterns of prechaos behaviour.

We again observe the hysteresis effect in very narrow frequency range $7.77\text{ rad/s} \le \omega \le 7.79\text{ rad/s}$, where transitional (prechaos) and (9,6)-periodic regimes are realising. In this frequency range the Poincaré map for transitional (prechaos) motion is becoming a set of points generally arranged in almost closed curve. It is the breakup of the quasiperiodic torus before the chaotic motion. At very narrow frequency range $7.80\text{ rad/s} \le \omega \le 7.815\text{ rad/s}$ there is a chaotic motion ($\lambda = 0.018$).

After that we see the zone with complicated motion picture under $7.80\text{ rad/s} \le \omega \le 7.89\text{ rad/s}$. Here chaotic motion alternates with prechaos and postchaos behaviour.

For example at Fig. 8 phase trajectories and Poincaré maps in such states are depicted.

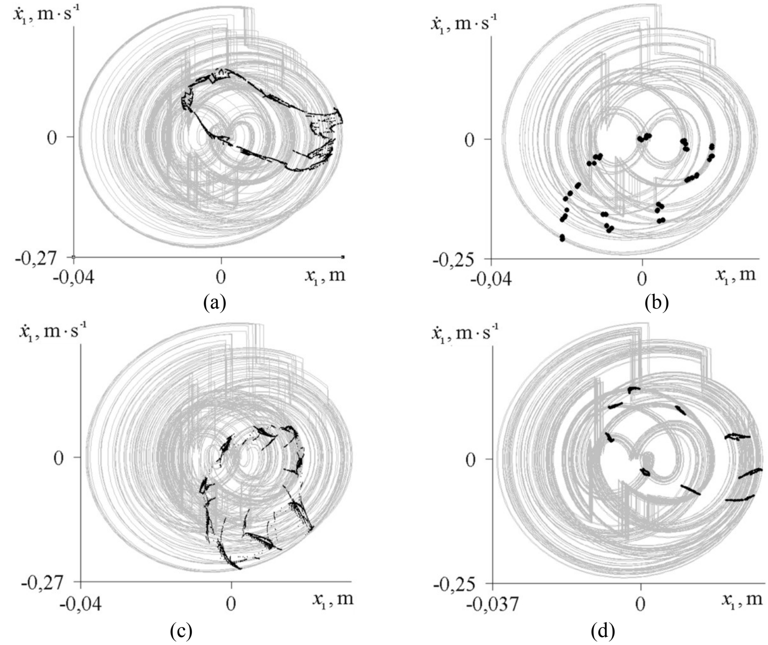
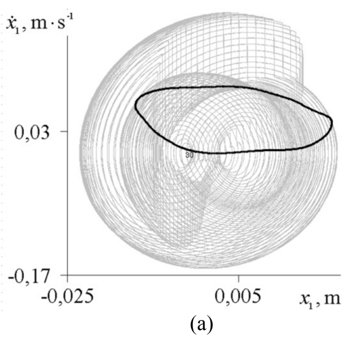


Fig. 8. Phase trajectories and Poincaré map for: (a) $\omega = 7.80$ rad/s , $\lambda = 0.018$; (b) $\omega = 7.82$ rad/s , $\lambda = 0.0092$; (c) $\omega = 7.83$ rad/s , $\lambda = 0.031$; (d) $\omega = 7.845$ rad/s , $\lambda = 0.0086$



Eventually there is really chaotic motion in narrow frequency range $7.90 \text{ rad/s} \le \omega \le 7.92 \text{ rad/s}$. At first we see how Poincaré map for quasiperiodic motion is deforming under $\omega = 7.93$ rad/s (Fig. 9) when frequency is decreasing. Chaotic motion is characterized by the breakup of the quasiperiodic torus structure as the control parameter is decreasing.

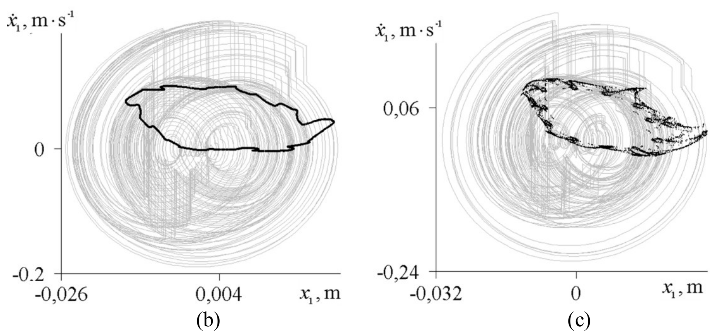


Fig. 9. Phase trajectories and Poincaré map for: (a) $\omega = 7.94$ rad/s , $\lambda = 0.0086$; (b) $\omega = 7.93$ rad/s , $\lambda = 0.0069$; (c) $\omega = 7.92$ rad/s , $\lambda = 0.014$

Let us have a look at chaotic motion under $\omega = 7.92$ rad/s , $\lambda = 0.014$. At Fig. 11 Poincaré map for main body m_1 and Fourier spectrum in logarithmic scale are depicted. Poincaré map does not consist of either a finite set of points or a closed orbit. Prof. F. Moon [1] have called his Poincaré map "Fleur de Poincaré" (Fig. 10). There is not the word "fleur" in English. There is word "Fleur-de-lis" which means "ірис" in Ukrainian. His Poincaré map is similar at fleur-de-lis. Prefix "de" is used in French, Poincaré was Frenchman. Therefore we call our beautiful map as "Leaflet de Poincaré" ("пелюстка" in Ukrainian).

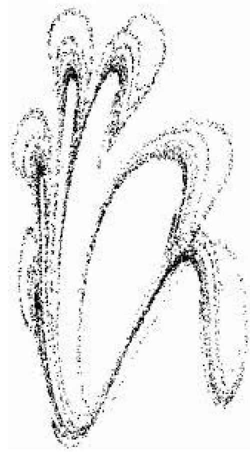


Fig. 10 Poincaré map – Fleur de Poincaré [1]

At Fig. 11 we see a broad continuous spectrum of Fourier components what is typical for chaotic motion. The generation of a continious spectrum of frequencies is one of the characteristics of chaotic vibrations.

Our vibroimpact system is the damped one. Poincaré map is the singular characteristic of chaotic vibrations in such system. The Poincaré map appears as an infinite set of highly organized points arranged similar to parallel lines. Chaotic motion is not a formless chaos but one in which there is some order that is fractal structure.

We enlarge a portion of the Poincaré map and observe further structure. We see that this structured set of points continues to exist after three enlargements (Fig. 12). So the motion appears to occur on the strange attractor. This embedding of structure within structure is a strong indicator of chaotic motion. It is similar to Cantor set.

We observe the fractal structure of Poincaré map depicted at Fig. 11, at least this structure looks highly fractal. We have been able to obtain it when we have had 207000 Poincaré points on the map. It is shown at Fig. 12. We think that just this fractal structure implies the existence of a strange attractor.

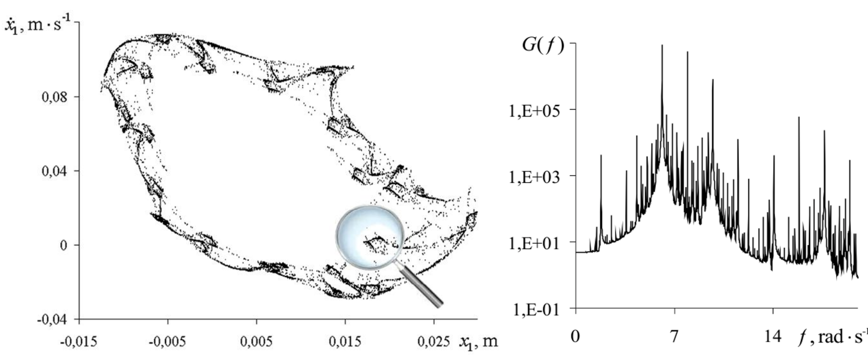


Fig. 11. Poincaré map ("Leaflet de Poincaré") and Fourier spectrum for ω=7.92 rad/s , λ = 0.014

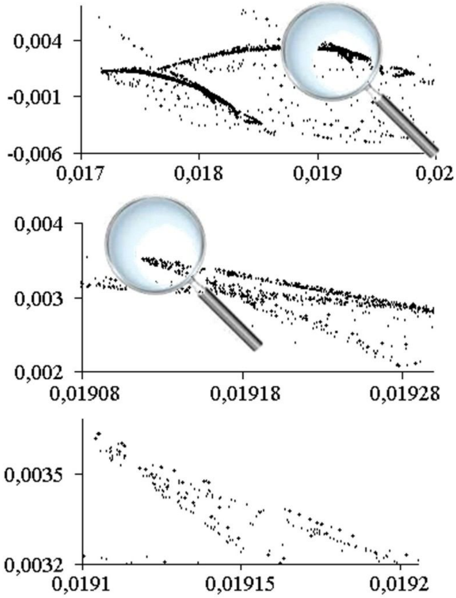


Fig. 12. Fractal structure for Poincaré map from Fig. 11

Thus Poincaré map, Fourier spectrum, the largest positive Lyapunov exponent, and fractal structure of Poincaré map confirm the chaoticity of this regime.

So we see three very short frequency ranges where chaotic motions are realizing:

$$7.80 \text{ rad/s} \le \omega \le 7.815 \text{ rad/s},$$

$$\omega = 7.83 \text{ rad/s},$$

$$7.90 \text{ rad/s} \le \omega \le 7.92 \text{ rad/s}.$$

We think these chaotic motions may be considered as transient chaos because chaotic vibrations appear for some parameter changes and then degenerate into a quasiperiodic motion after a short time [1, 29, 30].

4. Conclusions

Quasiperiodic route to chaos in vibroimpact system have been studied under changing of external frequency.

1. At this sufficiently complicated route different oscillatory regimes have succeeded each other many times under very small control parameter varying in narrow frequency range. There were periodic subharmonic regimes − chatters, quasiperiodic, and chaotic regimes. There were the zones of transition from one regime to another, the zones of prechaotic or postchaotic motion. The hysteresis effects (jump phenomena) have been observed at two frequency ranges.

2. The chaoticity of obtained regime has been confirmed by typical views of Poincaré map and Fourier spectrum, by the positive value of the largest Lyapunov exponent, and by the fractal structure of Poincaré map.

3. Our picture of quasiperiodic route to chaos confirm the theory by Newhouse, Ruelle, and Takens who suggested a new bifurcation scenario where a periodic solution produces subsequently a torus and then a strange attractor.

REFERENCES

1. *Moon F.C.* Chaotic vibrations: an introduction for applied scientists and engineers. – New York: Wiley, 1987. – С. 219.
2. *Thompson J.M.T., Thompson M., Stewart H.B.* Nonlinear dynamics and chaos. – John Wiley & Sons, 2002.
3. *Kuznetsov S.P.* Dynamical chaos //Moscow: Fizmatlit.-2006.-356P. – 2001.
4. *Schuster H.G.* Deterministic Chaos. An Introduction 2nd Revised Edition. – 1988.
5. *Luo A.C.J.* Analytical routes to chaos in nonlinear engineering. – John Wiley & Sons, 2014.
6. *Shvets A.Yu.* Deterministic chaos. Textbook, Kyiv, NTUU "KPI", 2010. http://chaos.kpi.ua/images/stories/Posibnik-nor.pdf (in Ukrainian)
7. *Volchenkov D., Leoncini X.* (ed.). Regularity and Stochasticity of Nonlinear Dynamical Systems. – Springer International Publishing, 2018.

8. *Volchenkov, D.*, "Survival under Uncertainty An Introduction to Probability Models of Social Structure and Evolution", Springer Series: Understanding Complex Systems, 240 pages, eBook ISBN 978-3-319-39421-3, ISBN 978-3-319-39419-0, Berlin / Heidelberg © 2016. http://www.springer.com/gp/book/9783319394190
9. *Grebogi C., Ott E., Yorke J.A.* Crises, sudden changes in chaotic attractors, and transient chaos //Physica D: Nonlinear Phenomena. – 1983. – Т. 7. – №. 1-3. – С. 181-200.
10. *Leine R.I., Van Campen D.H., Van de Vrande B.L.* Bifurcations in nonlinear discontinuous systems //Nonlinear dynamics. – 2000. – Т. 23. – №. 2. – С. 105-164.
11. *Kowalczyk P.* et al. Two-parameter discontinuity-induced bifurcations of limit cycles: Classification and open problems //International Journal of Bifurcation and Chaos. – 2006. – Т. 16. – №. 03. – С. 601-629.
12. *Manneville P., Pomeau Y.* Different ways to turbulence in dissipative dynamical systems //Physica D: Nonlinear Phenomena. – 1980. – Т. 1. – №. 2. – С. 219-226.
13. *Afraimovich V.S., Shilnikov L.P.* Invariant two-dimensional tori, their breakdown and stochasticity //Amer. Math. Soc. Transl. – 1991. – Т. 149. – №. 2. – С. 201-212.
14. *Shilnikov A., Shilnikov L., Turaev D.* On Some Mathematical Topics in Classical Synchronization.: a Tutorial //International Journal of Bifurcation and Chaos. – 2004. – Т. 14. – №. 07. – С. 2143-2160.
15. *Bakri T. Torus* Breakdown and Chaos in a System of Coupled Oscillators //International Journal of Non-Linear Mechanics. – 2005.
16. *Verhulst F.* Torus break-down and bifurcations in coupled oscillators //Procedia IUTAM. – 2016. – Т. 19. – С. 5-10.
17. *Komuro M.* et al. Quasi-periodic bifurcations of higher-dimensional tori //International Journal of Bifurcation and Chaos. – 2016. – Т. 26. – №. 07. – С. 1630016.
18. *Müller P.* C. Calculation of Lyapunov exponents for dynamic systems with discontinuities //Chaos, Solitons & Fractals. – 1995. – Т. 5. – №. 9. – С. 1671-1681.
19. *Stefanski A., Dabrowski A., Kapitaniak T.* Evaluation of the largest Lyapunov exponent in dynamical systems with time delay //Chaos, Solitons & Fractals. – 2005. – Т. 23. – №. 5. – С. 1651-1659.
20. *Andreaus U., Placidi L., Rega G.* Numerical simulation of the soft contact dynamics of an impacting bilinear oscillator //Communications in Nonlinear Science and Numerical Simulation. – 2010. – Т. 15. – №. 9. – С. 2603-2616.
21. *Bazhenov V.A., Pogorelova O.S., Postnikova T.G.* Stability and Discontinious Bifurcations in Vibroimpact System: Numerical investigations. LAP LAMBERT Academic Publ. GmbH and Co. KG Dudweiler, Germany. 2017.
22. *Bazhenov V.A.* et al. Stability and bifurcations analysis for 2-DOF vibroimpact system by parameter continuation method. Part I: loading curve //Journal of Applied Nonlinear Dynamics. – 2015. – Т. 4. – №. 4. – С. 357-370.
23. *Bazhenov V.A.* et al. Numerical Bifurcation Analysis of Discontinuous 2-DOF Vibroimpact System. Part 2: Frequency-Amplitude response //Journal of Applied Nonlinear Dynamics.– 2016. – 2016.
24. *Lamarque C.H., Janin O*. Modal analysis of mechanical systems with impact non-linearities: limitations to a modal superposition //Journal of Sound and Vibration. – 2000. – Т. 235. – С. 567-609.
25. *Shvets A.Y., Sirenko V.O.* Peculiarities of transition to chaos in nonideal hydrodynamics systems //Chaotic Modeling and Simulation.—2012.—2.—P. – 2012. – С. 303-310.
26. *Murguía J.S.* et al. Wavelet characterization of hyper-chaotic time series //Revista Mexicana de Física. – 2018. – Т. 64. – №. 3. – С. 283-290.
27. *Rubežić V., Djurović I., Sejdić E.* Average wavelet coefficient-based detection of chaos in oscillatory circuits //COMPEL-The international journal for computation and mathematics in electrical and electronic engineering. – 2017. – Т. 36. – №. 1. – С. 188-201.
28. *Bazhenov, V.A., Pogorelova, O.S., & Postnikova, T.G.* Lyapunov exponents estimation for strongly nonlinear nonsmooth discontinuous vibroimpact system. Strength of Materials and Theory of Structures, 2018, 99. (in press).
29. *Lai Y.C., Tél T.* Transient chaos: complex dynamics on finite time scales. – Springer Science & Business Media, 2011. – Т. 173.
30. *Afraimovich V.S., Neiman A.B.* Weak transient chaos //Advances in Dynamics, Patterns, Cognition. – Springer, Cham, 2017. – С. 3-12.

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РУЙНУВАННЯ ІНВАРІАНТНОГО ТОРУ У ВІБРОУДАРНІЙ СИСТЕМІ - ШЛЯХ ДО *КРИЗИ*?

Хаотичні коливання динамічних систем і сценарії їхнього переходу до хаосу –одне з найбільш цікавих і досліджуваних питань в нелінійній динаміці. Зокрема, сценарії переходу до хаосу в негладких динамічних системах представляють особливий інтерес. У цій статті ми вивчаємо квазіперіодичний шлях до хаосу нелінійної негладкої розривної віброударної системи с двома ступнями вільності. Руйнування інваріантного тора, або замкнутої кривої, має місце саме при квазіперіодичному переході до хаосу. Це дорога до *кризи*? У вузькому діапазоні частот різні коливальні режими багаторазово змінювали один одного при дуже малій зміні ведучого параметра. Це були періодичні субгармонічні режими (стук), квазіперіодичні та хаотичні режими, зони передхаотичного та післяхаотичного руху. Ефекти гістерезису (явища перекидання) виникали при збільшенні і зменшенні частоти. Спостережуваний хаос був перехідним. Хаотичність отриманого режиму підтверджувалася типовим видом відображення Пуанкаре і Фур'є спектру, позитивним значенням старшого показника Ляпунова та фрактальною структурою відображення Пуанкаре. Ці дослідження підтверджують теорію Ньюхауза, Рюеля і Такенса, які запропонували новий біфуркаційний сценарій, коли періодичне рішення народжує тор, а потім дивний аттрактор.

Ключові слова: віброударна система, динамічна поведінка, квазіперіодичні, хаотичні, субгармоніки, відображення Пуанкаре, спектр Фур'є, показник Ляпунова, фрактальна структура.

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РАЗРУШЕНИЕ ИНВАРИАНТНОГО ТОРА В ВИБРОУДАРНОЙ СИСТЕМЕ – ПУТЬ К *КРИЗИСУ*?

Хаотические колебания динамических систем и сценарии их перехода к хаосу – один из наиболее интересных и исследуемых вопросов в нелинейной динамике. В частности, сценарии перехода к хаосу в негладких динамических системах представляют собой особый интерес. В этой статье мы изучаем квазипериодический путь к хаосу нелинейной негладкой разрывной виброударной системы с двумя степенями свободы. Разрушение инвариантного тора, или замкнутой кривой имеет место именно при квазипериодическом переходе к хаосу. Это дорога к *кризису*? В узком диапазоне частот различные колебательные режимы многократно сменяли друг друга при очень малом изменении управляющего параметра. Это были периодические субгармонические режимы (стук), квазипериодические и хаотические режимы, зоны предхаотического и послехаотического движения. Эффекты гистерезиса (явления переброса) возникали при увеличении и уменьшении частоты. Наблюдаемый хаос был переходным. Хаотичность полученного режима подтверждалась типичным видом отображения Пуанкаре и Фурье спектра, положительным значением старшего показателя Ляпунова и фрактальной структурой отображения Пуанкаре. Эти исследования подтверждают теорию Ньюхауза, Рюэля и Такенса, которые предложили новый бифуркационный сценарий, когда периодическое решение рождает тор, а затем странный аттрактор.

Ключевые слова: виброударная система, динамическое поведение, квазипериодические, хаотические, субгармоники, отображение Пуанкаре, спектр Фурье, показатель Ляпунова, фрактальная структура.

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Bazhenov V.A., Pogorelova O.S., Postnikova T.G. **Invariant torus break-down in vibroimpact system – route to *crisis*?** // Strength of Materials and Theory of Structures: Scientific-and-technical collected articles – Kyiv: KNUBA, 2018. – Issue 100. – P. 3-17.
Quasiperiodic route to chaos in nonlinear non-smooth discontinuous 2-DOF vibroimpact system is studied.
Fig. 12. Ref. 30

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Баженов В.А., Погорелова О.С., Постнікова Т.Г. **Руйнування інваріантного тору у віброударній системі - шлях до *кризи*?**// Опір матеріалів і теорія споруд: наук.-тех. збірн. – К.: КНУБА, 2018. – Вип. 100. – С. 3-17.
Вивчається квазіперіодичний шлях до хаосу нелінійної негладкої розривної віброударної системи с двома ступнями вільності.
Іл. 12. Бібліог. 30 назв.

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Баженов В.А., Погорелова О.С., Постникова Т.Г. **Рразрушение инвариантного тора в виброударной системе – путь к *кризису*?**// Сопротивление материалов и теория сооружений: науч.-тех. сборн. – К.: КНУСА, 2018. - Вып. 100. - С. 3-17.
Изучается квазипериодический путь к хаосу нелинейной негладкой разрывной виброударной системы с двумя степенями свободы.
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