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Modeling of Crack Growth Process in Spatial Bodies Under Cyclic Loading Condition

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Summary. The algorithm for finite element modeling of fatigue cracks growth in the spatial bodies under cyclic loading using semianalytic finite element method (SFEM) is presented. The crack growth process is described by Paris' equation, stress intensity factor (SIF) is determined by the direct method. Testing of the algorithm is executed on the problem of the development of an elliptical crack in a prismatic body under the action of cyclic loading.

Key words: cyclic loading condition, crack, fracture mechanic, lifetime, spatial problem, semianalytic finite element method (SFEM).

INTRODUCTION

The determination of bearing capacity of responsible structure elements of different industries of technique needs for the taking into account of initial cracks presence. At the static loading a crack growth and further swift destruction takes place at exceeding fracture mechanics parameters of their critical values. The other mechanism of destruction is a result of crack growth under cyclic loading condition. A value of fracture mechanics parameters (stress intensity factor, SIF, in particular), arrived under external loading can be substantially less than critical one in this case, but cyclic influence of loading causes the gradual increase of crack. The life-time of structure element with a crack is considered outspent when a crack sizes acquires critical values. Thus, it is of interest in this connection to model a crack growth process under cyclic loading condition and to determine the amount of loading cycles to the achievement of critical size a crack.

The most well-known results of research of deformation of spatial bodies with cracks are limited of fracture mechanics parameters determination [1, 2, 6, 8, 10, 11] or with wide range exeperimental results of crack growth [12]. Moreover, there is some simplified and approximated methodologies for prediction of crack growth process: several results are known for plane (two-dimensional) problems [9, 13], three-dimensional problems has been considered using boundary elements method [7]. The more accurate results could be obtained using numerical techniques for stress-strain state analysis, in particular finite element method (FEM). Therefore the development of algorithms of crack growth process under the cyclic loading condition and it's realization using FEM is important problem.

PURPOSE OF WORK

The purpose of this paper is to highlight the main feature semianalytic finite element method (SFEM), of the numerical techniques of crack growth process modeling for spatial prismatic bodies under cyclic loading condition, which has been developed using SFEM, and to show an example of prognosis of shape and size changes of elliptical crack.

EQUATION AND METHODS OF ANALYSIS

Fracture mechanics relations. The crack growth process under the cyclic loading condition is characterized with the diagram of fatigue failure, that sets correlation between crack increment dl per number of loading cycle dN and change of SIF (or SIF increment ΔK). The most well-known approximation to this dependence is so-called Paris' relation [1, 12]:

$$\frac{dl}{dN} = C(\Delta K)^m , \qquad (1)$$

where: C, m – constants, that is determined by material, temperature, environment and other loading factors.

Dependence (1) has some limitation concerned with terms of loading, sizes of details and other. Not looking on it, use of (1) allows to solve a wide range of practical problem about crack growth and life-lime determination of responsible spatial structure elements [12].

Semianalytic finite element method (SFEM). The solution of spatial bodies deformation problems requires significant computational expences. The presence of crack increases it in times. Besides, a special algorithms for calculation of criteria fracture mechanic parameter (SIF in particular) and for crack growth process modeling are required. It is not always possible to solve these problems using modern powerful finite element software systems (ANSYS, ABAQUIS, NASTRAN etc.), based on traditional three-dimensional finite element problem definition.

SFEM is an effective instrument for numerical modeling of stress-strain state and deformation process of canonical form spatial bodies - inhomogeneous circle and, in particular, prismatic bodies (Fig.1). The term "inhomogeneous" is used in the sense of the variability of the physical properties along the forming. Being based SFEM, a discrete calculation model suggests the finite element mesh in the cross section of the examined object, and one finite element (FE) to be used in the orthogonal towards the cross sectional plane (along the forming, i.e. $z^{3'}$ coordinates). Thus, the FE size and configuration in the $z^{3'}$ direction is the same as the body one (Fig.2).

The main distinctive feature of SFEM is using of different approximation function in cross-section of the body (in plane $z^{1'} - z^{2'}$) and along $z^{3'}$ coordinates. Thus, the most universal representation of displacement using local FE coordinate system is:

$$u_{m'} = \sum_{S_1=\pm1}\sum_{S_2=\pm1} u_{m'(S_1S_2)}\left(\frac{1}{2}S_1x^1 + \frac{1}{2}S_2x^2 + \right.$$
$$\left. + S_1S_2x^1x^2 + \frac{1}{4}\right),$$

$$u_{s'} = \sum_{l=0}^{L} \bar{u}_{s'}^{l}\varphi^{(l)} ,$$

where: $\varphi^{(l)}$ - is the coordination function systhems, presented with Laugrange-Michlin polynoms.

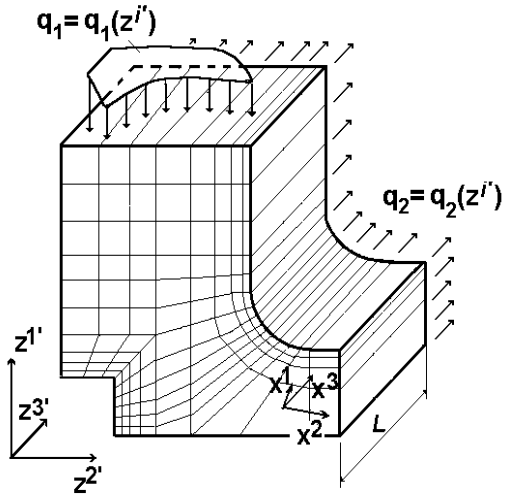

Fig. 1. Prismatic inhomogeneous body

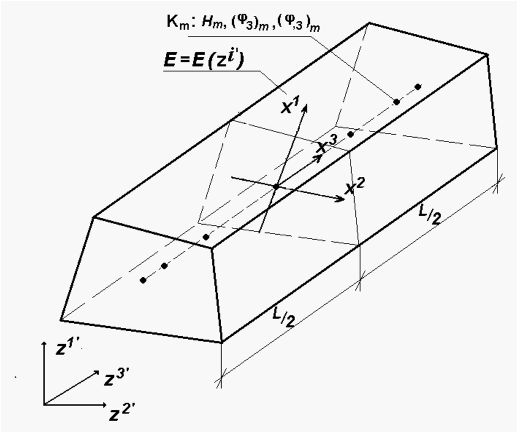

Fig. 2. Prismatic inhomogeneous finite element

The stress-strained state parameters values are calculated in integration point K_m. along $z^{3'}$ coordinates. The quantity of integration point depends of heterogeneity distribution of stress-strain parameters along $z^{3'}$ coordinates and determined on the basis of study of the convergence of obtained solution.

SFEM allows significantly reduce the computational expenses for solving of spatial problem, particularly on the stages of stiffness matrix calculating and FEM linear equations systems solving. The efficiency and accuracy of the method is shown for a wide range of linear and nonlinear problems of mechanics [3-5], where readers can also find a more detailed description of the method features, its implementation and links to additional author's publications.

FINITE ELEMENT ALGORITHMS FOR FRACTURE MECHANIC'S PROBLEM SOLUTION

At the numerical decision of a crack growth problem under the cyclic loading condition the loading process presents with the sequence of steps after the cycles of application of the external loading. Corresponding discrete presentation of equation (1) for description of cracks growth has a next kind:

$$\frac{\Delta l}{\Delta N} = C(\Delta K)^m, \tag{2}$$

where: Δl – an increase of characteristic crack's sizes in the certain point of front for the amount of cycles of loading ΔN.

At implementation of numeral integration of (2) provides calculation of follow values at each step:

- the SIF value $K_I(l^i)$ in each point of crack front on the basis of results of the stress-strained state determination of body with a crack:
- the corresponding values of increase of characteristic sizes of crack after ΔN cycles in every point of front *i (i=1..k):*

$$\Delta l_m^i = C\left(K_I(l^i)\right)^b \Delta N_m. \tag{3}$$

- the characteristic sizes of crack l_m^i at every step *m* using sizes of crack on a previous step l_{m-1}^i taking into account their increases Δl_m^i:

$$l_m^i = l_{m-1}^i + \Delta l_m^i. \tag{4}$$

- the new coordinates of nodes of crack front are and of other nodes of FE model.

Consider the above procedure for the case of three-dimensional body.

In case of the spatial stress-strained state the curvilinear front of crack (shown on Fig.3 by a thick solid line) is approximated by the segments of polygon (shown on fig.1 by a stroke line), that consistently connect the nodes of discrete model, that located on the crack's front. Amount of this nodes is determined on the basis of convergence of numeral decision of problem about the stress-strained state of body with a crack and achievement of necessary exactness of determination of SIF distribution along front of crack.

In case of consideration of curvilinear cracks front, the SIF values and increases of characteristic sizes of crack, that is calculated on a formula (3), are variables along front. Accordingly, at every solution step the configuration of crack front changes.

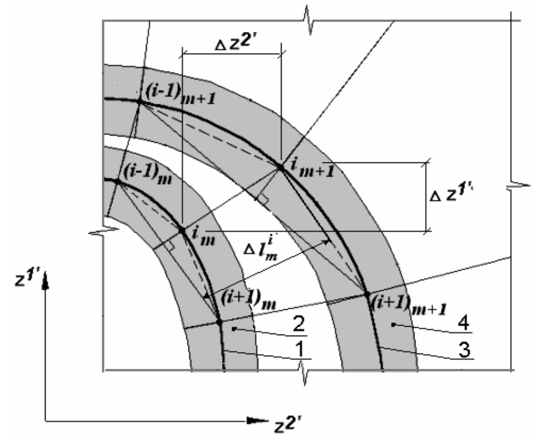

Fig. 3. The FE element discritization of the crack front and coordinates of front points: front of crack (1) and near-tip area (2) on the step of *m*; front of crack (3) and near-tip area (4) on a step *m*+1

A calculation of the SIF value $K_I(l^i)$ executed by a direct method. It provides to use of obtained with finite element solution stress and displacement distribution near crack front (tip at two-dimensional case). The stress and displacement components, being oriented along the normal to the surface (front of crack) used in the most commonly sold type of fracture - normal separation crack, or crack of type I, Fig.4 [8].

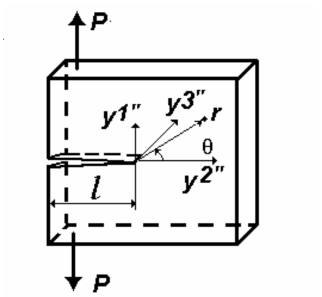

Fig. 4. Crack of type I (the normal separation crack)

SIF calculation executed separately from the values of stress $K_I(\sigma)$ and displacement $K_I(u)$ using well-known dependences :

$$K_I(\sigma) = \frac{\sigma^{1"1"}\sqrt{2\pi r}}{\cos\frac{\theta}{2}\left(1+\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)}$$

$$K_I(u) = \sqrt{\frac{2\pi}{r}}\,\frac{u_{1"}\,G}{\sin\frac{\theta}{2}\left(2-2G-\cos^2\frac{\theta}{2}\right)}, \quad (5)$$

where: r, θ – point (nodes) coordinates (Fig.4).

SIF calculations executes within the limits of near-tip area of square form, with topological sizes of 6x6 FE. The size of FE accepted as 1/10 of characteristic size of crack l_{cr}. The half of the marked near-tip area is examined for the case of the normal separation crack as a result of symmetry of distribution of stress-strain parameters in relation to the surface of crack. The description of crack location executed with boundary condition - absence of displacements in the nodes on plane of symmetry. Thus in prismatic bodies with transversal cracks (surface of crack is normal to $z^{3'}$ coordinates) the size of near-tip area in direction, normal to the surface of crack is equal to $0,3\,l_{mp}$ (Fig.5).

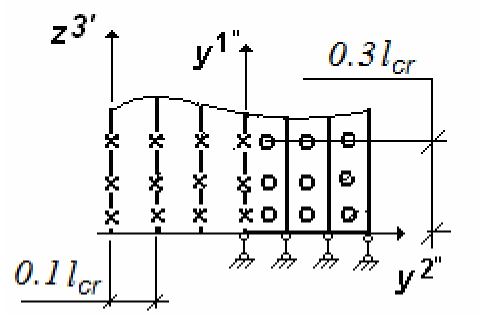

Fig. 5. The near-tip area for SIF calculation

SIF has been calculated after displacements ($K_I(u)$) in part of area, that borders from a crack surface (in points, marked by crosses on Fig.5). SIF has been calculated after stresses ($K_I(\sigma)$) in part of area, that that is located after front of crack (in points, marked by rounds on Fig.5).

Within the limits of each of the marked parts of near-tip area a mean SIF value after tensions $K_{I\,\mathrm{av}}(\sigma)$ and after displacements $K_{I\,\mathrm{av}}(u)$ are determined. Then this two values averaged in turn for determination resulting SIF value K_I:

$$K_I = \frac{K_{I\,\mathrm{av}}(\sigma)+K_{I\,\mathrm{av}}(\mathrm{u})}{2}. \quad (6)$$

At consideration of body with transversal crack SIF calculation is conducted in a certain amount of points along crack front. Their location coincides with nodes of FE model. Since SIF after displacements are calculated at the nodes (for example, crossection 1-1, Fig.6), and SIF after stresses - in the center of finite element (crossections 1’-1’ i 2’-2’, Fig.6), it is necessary to account for SIF after stresses in the two adjacent to FE nodes. This procedure is illustrated in Fig. 6 and with the following formulas:

$$\left[K_{I\,\mathrm{av}}(\sigma)\right]_1 = \left[K_{I\,\mathrm{av}}(\sigma)\right]_{1'} \qquad (7)$$

$$\left[K_{I\,\mathrm{av}}(\sigma)\right]_2 = \frac{\left[K_{I\,\mathrm{av}}(\sigma)\right]_{1'} + \left[K_{I\,\mathrm{av}}(\sigma)\right]_{2'}}{2} \qquad (8)$$

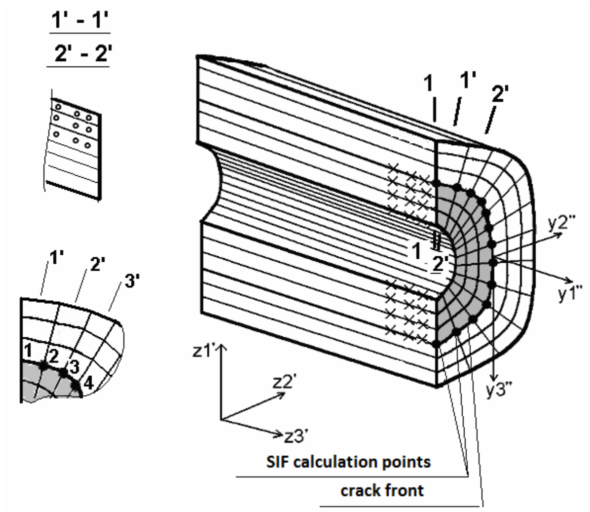

Fig. 6. Points of SIF calculation prismatic bodies

Calculation of coordinates $(z^{k'})^i_m$ of node i of crack front on the step m executed after the values of increases of crack length Δl^i_m (3) in front points after next formulas (Fig.3):

$$(z^{k'})^i_{m+1} = (z^{k'})^i_m + (\Delta z^{k'})^i_m,$$
$$(\Delta z^{k'})^i_m = \Delta l^i_m \cos(\phi^{k'})^i_m \ , \qquad (9)$$

where: $\phi^{k'}$ – a corner is between direction to the axis $z^{k'}$ and by direction of movement of points of crack front:

$$\phi^{1'} = \pi/2 - \alpha = \pi/2 - arctg\left(\frac{(z^{2'})^{i-1}_m - (z^{2'})^{i+1}_m}{(z^{1'})^{i+1}_m - (z^{1'})^{i-1}_m}\right)$$
$$\phi^{2'} = \phi^{1'} - \pi/2 \,. \qquad (10)$$

The change of all nodes of near-tip area executed on identical dimensions, calculated by after correlations (9), (10) along the line, that is normal to the front of crack. This line is passes through a point of front. It allows to save the characteristic sizes of FE of near-tip area during a crack increase.

The change of location of nodes, that lie outside of near-tip area on the same distance in the case of modeling of crack growth in the finite size bodies leads to the formation of degenerate FE at the boundary of the body, as illustrated in Fig.7,a. It was suggested to overcome this problem, to displace of nodes of a discrete model within near-tip area size on crack increment and to displace nodes outside of near-tip area to a distance that decreases linearly inverse-proportional to the distance from each node to the crack tip (Fig. 7,b). This method is more complex in terms of implementation, but it allows to get rid of the problem of the formation of degenerate FE.

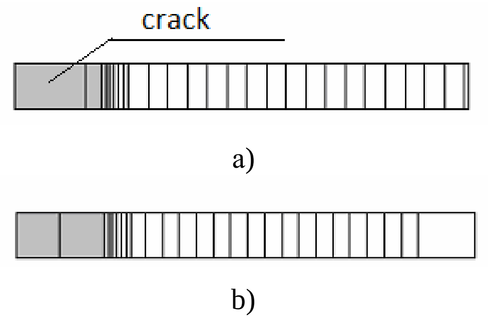

Fig. 7. Type of fragment of discrete model after crack growth: at the displace all the nodes on identical distances (a); at the displace the nodes on different distances with application of linearly inverse-proportional reduction (b)

RESULTS OF FINITE ELEMENT MODELING OF FRACTURE DUE TO CRACK GROWTH

The approbation of algorithm of crack growth modelling in spatial bodies was conducted on an example about growth of initial elliptic crack in an endless prismatic body under the action of the cyclic loading. (Fig.8, a=0.6, b=0.4).

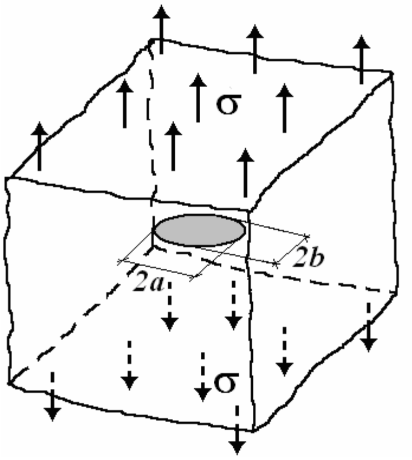

Fig. 8. An endless body is with an elliptic crack

As the examined object has three planes of symmetry, a discrete model is built for a 1/8 part of body (Fig.9).

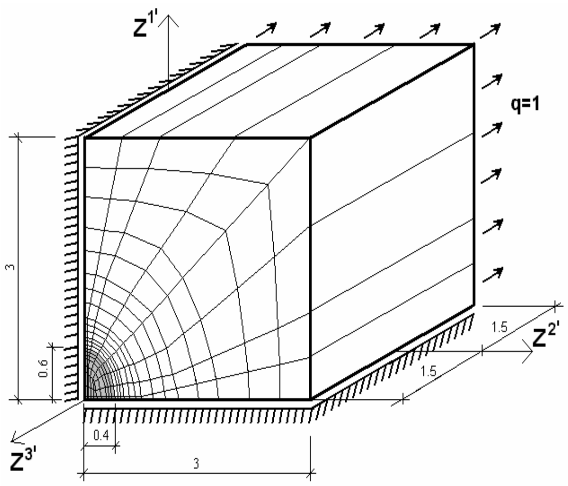

Fig. 9. A discrete model of SFEM for an endless body with an elliptic crack

The distribution of SIF along the front of initial crack, obtained with consideration of SFEM solution convergence on quantity of FE in cross-section mesh and on quatity of polynoms $\varphi^{(l)}$ in displacement distribution, is snown on Fig.7. It well comports with standard values of with well-known analytical decisions [6]. Thus, it could be expected, that firther modeling of crack growth would be correct.

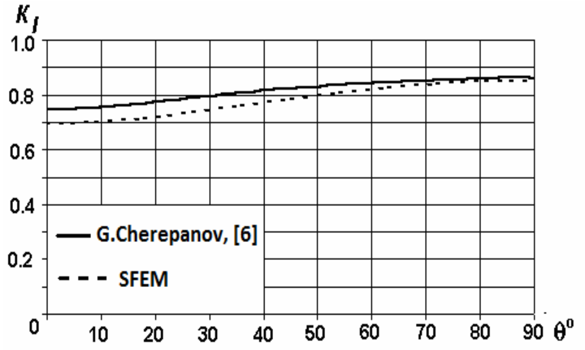

Fig. 10. Distribution of SIF along the initial front of crack

The values of constant of Paris' equation for crack growth description were accepted the next: $b = 4$, $C = 1.63\times10^{-10}$, that correspondents to stell in normal temperature condition.

During realization of algorithm of crack growth modeling two alternate variants of changing of configuration of discrete model was studied. According the first one it is considered, that crack growth at every step takes place in ortogonal direction to current configuration of crack front. Realization of this supposition on a discrete model suggested, that the value of crack increases Δl_m^i is put aside along a line, which is ortogonal to the segment, that connects $(i\text{-}1)_m$ and $(i+1)_m$ points of front. According to the second variant crack growth at every step considered after a perpendicular to front of initial crack. In this case the value of crack increases Δl_m^i in points front is put aside along a line, ortogonal to the segment that connects points *i-1 and i+1* of initial front of crack.

Verification of authenticity of application of foregoing suppositions was made on the basis of analysis of results convergence at successive reduction to the step after the some amount of loading cycles and on it coincidence of final result with the data given in [6].

Results testify that convergence of step-by-step algorithm of problem solution depending after the size of step *ΔN* is more better at application of the first variant. The difference between the characteristic sizes of crack, calculated after 24 and 48 steps of problem solution, which corresponds

30×10^8 cycles of loading is not significant and folds 2.6% for the first variant and 1.1% for second variant. However, the final characteristic sizes of crack after 48 steps differ on 14% for two variants. Dependence of error of length of crack calculation at M steps of problem solution in relation to the characteristic sizes of crack, certain at 48 steps for the front points, that is located on lines along axes $z^{1'}$ ($\theta=0^\circ$), $z^{2'}$ ($\theta=90^\circ$) for the first variant shown on a Fig 11. The general view of configuration of crack front and near-tip area after 30×10^8 cycles by comparison to initial, shown on Fig.12.

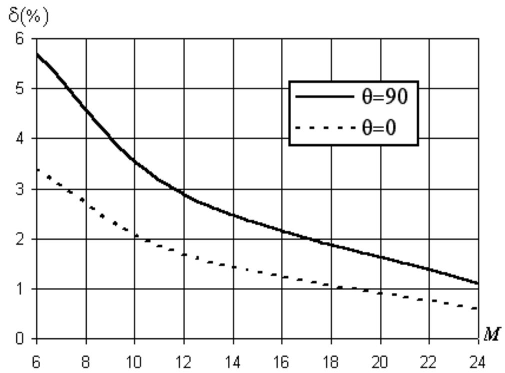

Fig. 11. A relative error of length of crack calculation

As as can be seen from Fig.12 the selection of algorithm significantly affects not only the quantitative growth of the crack (the number of cycles and the characteristic sizes) but also on the final configuration of the front. According to work [6] by conclusions, growth of initial elliptic crack will come true so that configuration of front will head for a circle. The marked reasoning is fully confirmed that the modeling of crack growth in every point should be in a direction orthogonal to current configuration of front (Fig.12, a). In another case, configuration of front crack takes shape of ellipse, prolonged in direction orthogonal to the initial location of front of crack (Fig.12, b).

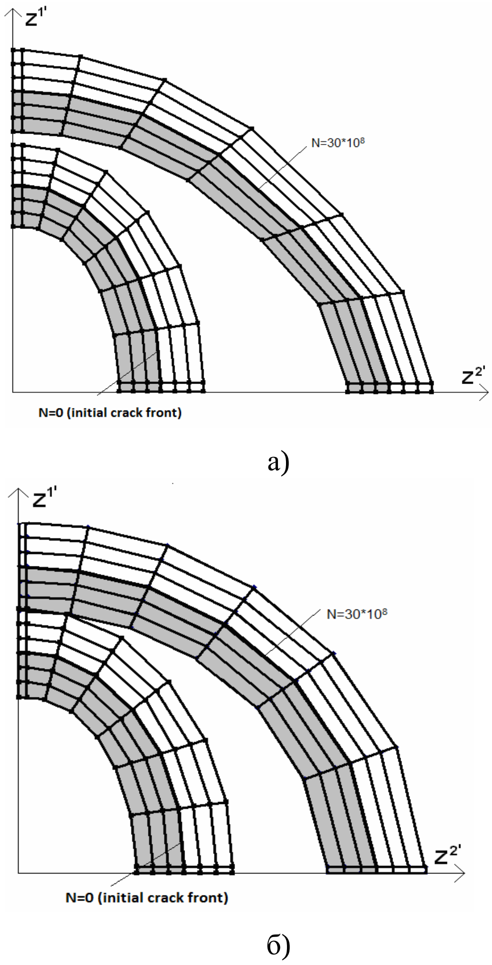

Fig. 12. Configuration of crack front and near-tip area of discrete FE model at application of the first (a) and second (b) variants of algorithms

Further decision of problem has shown, that the crack front in future transformed to circle shape and the process of crack growth occurs rapidly (Fig. 13).

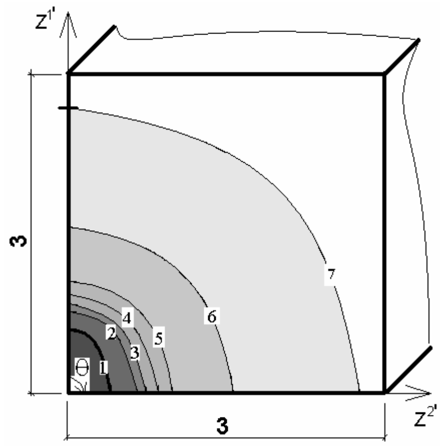

Fig. 13. Configuration of crack front after N of loading cycles: 1- initial crack (N=0), 2 – N=16*10^8; 3– N=20*10^8; 4– N=24*10^8; 5– N=32*10^8; 6– N=48*10^8; 7– N=56*10^8

CONCLUSIONS

1. The developed algorithm of restructuring of FEM discrete model provides the ability to reliably modeling crack growth. Numerical results of solving the test problem coincides with the calculated results of other authors.

2. Applied assumptions about the development of cracks in a direction orthogonal to its current configuration dozvolyayuye adequately simulate the process of crack growth in spatial bodies which are characterized by a curved crack front configuration and variability values SIF along the crack front.

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МОДЕЛИРОВАНИЕ РАЗВИТИЯ ТРЕЩИН В ПРОСТРАНСТВЕННЫХ ТЕЛАХ ПРИ ЦИКЛИЧЕСКОЙ НАГРУЗКЕ

Аннотация. В статье представлен алгоритм конечноэлементного моделирования развития усталостных трещин в пространственных телах при циклическом нагружении. Механизм развития трещины описывается уравнением Пэриса. Коэффициенты интенсивности напряжений на каждом шаге задачи определяются прямым методом. Апробация алгоритма выполнена на задаче о развитии эллиптической трещины в призматическом теле под действием циклической нагрузки.

Ключевые слова: циклическая нагрузка б трещина, механика разрушения, ресурс, пространственная задача, ароуаналитический метод конечных элементов (ПМКЭ).