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BUCKLING OF THE STEEL FRAMES WITH THE I-SHAPED CROSS-SECTION COLUMNS OF VARIABLE WEB HEIGHT

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Abstract. Presented is the research of the stability of portal frames made of variable I cross-sections, depending on supports fastening factors and frames elements unfastening. In the process of mathematical research examined were five different cases of fixing columns nodes of portal frames resiliently mounted in each case, the stability criteria having been defined. In addition, conducted were studies to determine the coefficients of the portal frames elements effective length calculation in finding critical load on the column. Coefficients of the effective length factor of the welded variable I cross-section columns have been obtained. The influence of brace systems on stiffening of the whole structure, stability of the unit frames as well as the overall stability of the building with computer simulation and calculation have been studied, the coefficients of the influence of the frame structure on the stability of the unit frames having been obtained.
Keywords: variable cross-section; elastic fixed-support; effective length factor, portal frame; buckling of unit frame, load factor.

Introduction

The buckling of members of portal frame undoubtedly belongs to the important problems of designs of steel constructions [1, 53, 54, Timoshenko, S.P. (1908)]. The variable cross-section columns are an effective element. Therefore, today the research of stability loss and the theoretical research of Stability analysis of tapered elements should be more extensive. It is necessary to obtain more numerical examples of critical buckling load and to develop methods of calculating effective length factor.

Portal frames members buckling undoubtedly belongs to the range of important problems of steel constructions design [1, 5, 6, 9, 11, 12, 14, 15, 25] and [33, 34, 35, 38, 45, 46, 48, 57, 58]. The variable cross-section columns are an effective element.

Therefore, development of a consistent buckling design procedure for tapered columns is great importance [28, 29, 41]. The first works of studying buckling analysis of elements of variable cross-section were written by Dynnyk A. and Morley A. [17, 18, 19, 34, 46]. A. Dynnyk has reduced the governing differential equation for buckling columns with variable cross-section to the linear differential equations differential equation with variable coefficients. The solution of the governing equation is obtained due to Bessel functions by Gringhila method (The applications of elliptic functions (London, 1892). The main results of these studies were translated in English by Malets (1925) [34].

Today the Bessel functions used to solve problems of loss stability of tapered elements are well known [6, 7, 8]. Out-of-rotation plane bending vibrations of a rotating tapered beam with periodically varying speed are presented in work [13]: "the integro-partial differential equation of the beam is discredited via Galerkin's method and a set of ordinary differential equations with periodic coefficients (Mathieu–Hill type equations) is obtained".

An approximate method was proposed for analyzing the problem of beams of variable cross-section in article [4, 20, 42].

Articles [21, 22, 23, 24, 25] present a number of stability problems for columns and simple frames that have a post and a variable cross-section of non-uniform members. A free vibration of axially functionally graded beams with non-uniform cross-section studies in [30]. Buckling analysis of non-uniform and axially graded columns with varying flexural rigidity present in [3, 31, 35, 36, 39, 40, 44, 48]. The new numerical method is proposed [34] for the dynamic and stability analysis of elastic plane structures consisting of beams with constant width and variable depth. In this article [37] a generalized finite element for buckling analysis of tapered columns with various cross sections is established by using Chebyshev polynomial approach to the governing differential equation. The heterogeneous prismatic finite element with variable cross sectional area and taking into account the variability of components of metric tensor are presents in [10], prismatic finite element used for studies and analysis of non-uniform elements. In article [2] the solution of an ordinary differential equation of the fourth order with variable coefficients is used the approach using power series is given. The exact elastic stability functions for any general non-prismatic beam-column element with a uniform tensile or compressive axial force are obtained.

Bazeos, N. and Karabalis, D.L. [11] developed the approximate method for quick calculation of the critical load of tapered columns. The method is based on a series of dimensionless design-oriented charts related the critical load of linearly tapered columns of I-section to the taper ratio and boundary conditions.

Coşkun, S.B. and Atay, M.T. use variation iteration method for research critical buckling load for elastic columns of constant and variable cross-sections [16]. The work of Huang Y. and Li X.F. authors have reduced the governing differential equation for buckling of columns with varying flexural rigidity to Fredholm integral equation [31].

In the work [39] the buckling of a non-uniform column with spring supports under combined concentrated and distributed loads is presented. The governing equation for buckling of a one-step non-uniform column is reduced to Bessel equations and other solvable equations for 13 cases, several of which are important in engineering practices.

In the method [43] Ozay, G. and Topcu, A. proposed a general stiffness matrix for non-prismatic members that is applicable to Timoshenko beam theory has been derived. The stiffness coefficients have been determined for constant, linear, and parabolic height member's variations, employing analytical and numerical integration techniques.

Rezaiee-Pajand M., Shahabian F., Bambaeechee for simple frames presented methodology to determine critical load and effective length factor for buckling of a frame with tapered and prismatic columns [46]. The combined effect of the shape factor, taper ratio, elastic bracing system, and joint flexibility on the critical buckling load, buckling length factor of portal steel frames are considered.

In article [47] the method calculating of the critical buckling load of portal frames consisting of linearly tapered members is presented. Values of the factor of the estimated length of columns with fixed supports of portal frames The factor of the estimated length of the columns of the portal frame with hinged supports is obtained.

Elastic buckling loads of columns with variable cross-section has been studied due in the works [49, 50, 51, 52, 55].

In the research of Wei, D.J., Yan, S.X., Zhang, Z.P., and Li, X.F. [59] of critical load for buckling of non-prismatic columns under self-weight and tip force the governing equation subject to associated boundary conditions is transformed into an integral equation. Critical buckling load is then determined as the lowest value of the resulting integral equation.

A new shape function for tapered three-dimensional beams with flexible connections was obtained due to the analysis of Valipour, H.R. and Bradford, M.A. [56].

Non-linear post buckling analysis of frames and columns with was made in the works [5, 60, 61].

The effects of shear deformations are taken into account in the stability analysis for variable cross-section columns based on Timoshenko theory and the energy method in theoretical analysis [1, 10, 55].

These researches of rods buckling were conducted due to L. Euler's, the first work [26].

Foundation of the problem. Defining sustainability criteria and coefficients of the effective length of the elements of portal frames, taking into account elasticity of supports when calculating a flat buckling are important issue in designing of steel construction.

Methods of research. To research the buckling criteria of portal frames and calculating the coefficients of the effective length of columns mathematical modeling method was used. General view of the structure model is shown in Figure 1.

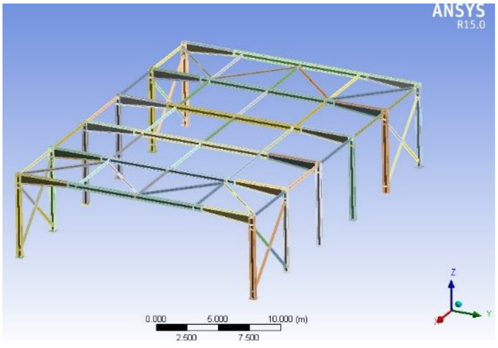

Fig. 1. General view of the model

Results. In Figure 2 was considered a portal frame (Figure 1) buckling resistance with the columns of I-shaped cross-section with variable web height and the girder with constant cross-section. The frame has a span – ***L*** and column height – ***l***. The column stiffness of *I*-shaped cross-section with variable web height considerably precise is written by the parabolic regularity if the correlation of least cross-section moment inertia I_{xn} to the largest cross-section moment inertia I_{x0} is situated in diapason $I_{xn}/I_{x0} = 0.1…0.95$.

Five cases of buckling column with variable cross-section on elastic support were studied.

Function $I_{xz} = I_x(z)$ is the moment of inertia of the variable cross-section is approximation tapered columns with I-section for steel portal frames. The I_{xz} of the columns has approximation by the following parabolic function.

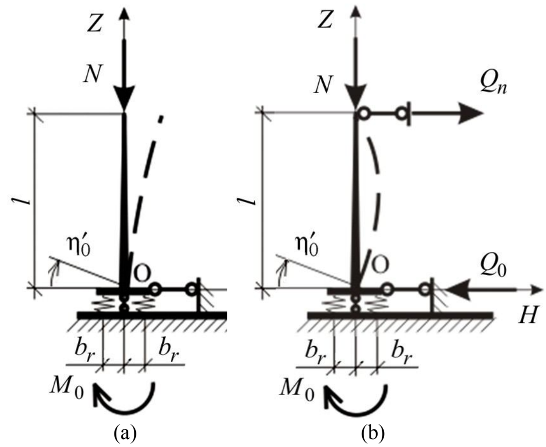

Fig. 2. Cases of resistance centrally compressed rods with variable cross section on elastic supports: (a) - case 1; (b) - case 2

$$I_{xz} = I_x(z) = I_{x0}(1-\gamma_y t_z)^2; \quad \gamma_y = 1 - I_{xn}/I_{x0}, \tag{1}$$

where I_{x0} – is a maximum moment of inertia of variable cross-section of member with coordinate $z_0 = 0$, I_{xn} – is moment of inertia with of minimum section dimensions, which has coordinate $z_n = l$.

We took the hypothesis: deformation of the column is satisfactorily described by the Bernoulli–Euler theory [53], and was acceptance the assumption that the load is applied only at the nodal points.

The bending moment at any cross section is

$$(\eta_z - \eta_0)N + M_{x0} - M_{xz} + \frac{Q_0 l}{N}\frac{z}{l} = 0, \tag{2}$$

where M_{xz} – bending moment at any cross section, M_{x0} – bending moment at cross section with coordinate $z_0 = 0$, N – is a constant axial compressive load, Q_0 – shear force at cross section with coordinate $z_0 = 0$, η_0 – displacement (deflection) cross section with coordinate $z_0 = 0$, η_z – displacement any cross section.

Governing equation of flexural deformation of the column of variable cross-section may be written as (of the member is approximated by the following parabolic function).

$$\eta''(1-\gamma_y t_z)^2 + \eta_z\frac{Nl^2}{EI_{x0}} = \frac{Nl^2}{EI_{x0}}\eta_0 - \frac{M_{x0}l^2}{EI_{x0}} - \frac{Q_0 l^3}{EI_{x0}}t_z, \quad t_z = \frac{z}{l}, \quad k^2 = \frac{Nl^2}{EI_{x0}}, \tag{3}$$

$$\eta_z = \eta_0 - \sqrt{B_z}\frac{l}{v\gamma_y}\eta_0' \sin(vu_z) + \left(\frac{M_{x0}l^2}{k^2 EI_{x0}}\left\{\sqrt{B_z}\left[-\frac{1}{2v}\sin(vu_z) + \cos(vu_z)\right] - 1\right\} - \right.$$

$$\left. - \frac{Q_0 l^3}{k^2 EI_{x0}}\left[\sqrt{B_z}\frac{1}{v\gamma_y}\sin(vu_z) + t_z\right], \quad \sqrt{B_z} = \sqrt{1-\gamma_y t_z}, \right.$$

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$$u_z = \ln(1-\gamma_y t_z),\ u_n = \ln(1-\gamma_y),\ v^2 = k^2/\gamma_y^2 - 0,25,\ v^2 + 0,25 = k^2/\gamma_y^2. \quad (4)$$

This homogeneous linear differential equation is second-order differential equations with variable coefficients. The general solution for these differential equations has analytical solution [6, 7, 8, 17, 18].

The general solution (4) written in form of method of initial parameters.

Was made studies for 5 cases stability of elastic rods with variable cross-section where columns have elastic support.

Case 1 (Fig. 2(a)). Column has elastic fixed-support for cross section with maximum dimensions (z=0) and free-end for cross section with minimum dimensions (z=l).

Boundary conditions are:

$$M_{xn} = 0,\ Q_n = Q_0 = 0,\ \eta_0\eta = 0,\ M_{x0} = -\frac{k_r b_r^2}{2}\eta_0', \quad (5)$$

where b_r – the width of the base of the column, and k_r – the coefficient of rigidity of elastic fixed base, η_0' – the rotation angle of cross section of elastic fixed–support.

The factor of elastic fixed-support may be written as:

$$b_{Er} = (EI_{x0}/l)(\eta_0'/M_x) = EI_{x0}/(0,5lb_r^2k_r). \quad (6)$$

The combination of boundary conditions (5) and the decision (4) provides stability loss equation to calculate the critical buckling load of column with varying cross-section (7).

$$\left(k^2 b_{Er} - \frac{\gamma_y}{2}\right)\cdot\frac{\mathrm{tg}(vu_n)}{v\gamma_y} + 1 = 0. \quad (7)$$

Equation of stability (7) makes it possible to determine the stability factor and coefficient of effective length. If $k_r \to\infty$; $b_{Er} \to 0$; we have boundary conditions for column with varying cross-section with fixed support – free-end. Stability loss equation (7) gives stability loss equation for column with fixed-support – free-end

$$\frac{\mathrm{tg}(vu_n)}{2v} - 1 = 0. \quad (8)$$

For variable cross-section column with fixed support – free-end were obtained factors effective length in Table 1.

Table 1

Effective length factor for variable cross-section column with fixed-elastic support and free-end. Case 1, (Fig. 2(a))

I_{xn}/I_{x0}	0,99	0,70	0,50	0,30	0,20	0,10
μ_x, b_{Er} =0	2,0003	2,107	2,209	2,366	2,491	2,704
μ_x, b_{Er} =0,033	2,0669	2,169	2,268	2,419	2,54	2,747
μ_x, b_{Er} =0,33	2,635	2,708	2,779	2,891	2,983	3,145
μ_x, b_{Er} =0,5	2,918	2,981	3,043	3,141	3,223	3,365
μ_x, b_{Er} =1,0	3,652	3,699	3,746	3,819	3,879	3,985
μ_x, b_{Er} =2,0	4,809	4,843	4,876	4,928	4,971	5,046

Case 2 (Fig. 2(b)). Column has elastic fixed-support for cross section with maximum dimensions (z=0) and pin-ended (articulated) support for cross section with minimum dimensions (z=l).

Boundary conditions are:

$$M_{xn} = 0,\ \ Q_n = Q_0,$$

$$\eta_0 = \eta_n = 0,\ \ Q_0 l = 0,5 k_r \eta_0' b_r^2,\ \ Q_0 l = -M_{x0},\ \ M_{x0} = -0,5 k_r \eta_0' b_r^2. \quad (9)$$

Stability loss criterion to calculate the critical buckling load for column with varying cross-section may be written:

$$(k^2 b_{Er} - \frac{\gamma_y}{2} + 1)\frac{\operatorname{tg}(\nu u_n)}{\nu \gamma_y} + 1 = 0\,. \quad (10)$$

Effective length factor for variable cross-section column with fixed-elastic support and articulated–end is in table 2.

Table 2

Effective length factor for variable cross-section column with fixed-elastic support and articulated–end

I_{xn}/I_{x0}	μ_x, b_{Er} =0	μ_x, b_{Er} =0,5	μ_x, b_{Er} =1,0	μ_x, b_{Er} =2,0
0,99	0,7009	0,9247	0,9584	1,0023
0,90	0,7177	0,9454	0,9804	1,0263
0,80	0,7389	0,9713	1,008	1,0564
0,70	0,7633	1,0011	1,04	1,0911
0,60	0,7921	1,0361	1,077	1,1319
0,50	0,8271	1,0782	1,122	1,1811
0,40	0,8711	1,1307	1,178	1,2428
0,30	0,9299	1,2002	1,252	1,3245
0,20	1,0164	1,3009	1,359	1,4434
0,10	1,1738	1,4802	1,5503	1,6555
0,01	1,7661	2,1179	2,2225	2,401

Case 3 (Fig. 3(a)). Column has fixed support for cross section with maximum dimensions (z=0) and articulated elastic support for cross section with minimum dimensions (z=l). η_n – displacement cross section with coordinate z_n=l, displacement of articulated elastic support. Coefficient of rigidity of elastic articulated support is k_{r3}. Boundary condition are the coefficient of rigidity of elastic fixed-base.

$$\eta_0 = \eta_0' = 0,\ M_{xn} = 0,\ \eta_n \neq 0,\ \ Q_0 = Q_n = -\eta_n k_{r3}. \quad (11)$$

Standard procedures connections equation Boundary condition and the general solution give the stability loss equation (criterion) to calculate the critical buckling load of column with varying cross-section. By using factor of elastic articulated support (6) – b_{Er}, was obtained the stability loss criterion:

$$\left[\frac{1}{(1 - k^2 / b_{E3})} - \frac{\gamma_y}{2}\right]\frac{\operatorname{tg}(\nu u_n)}{\gamma_y \nu} + 1 = 0\,. \quad (12)$$

In table 3 effective length factor for variable cross-section column with fixed-support and elastic articulated support for $b_{E3} = 0 \ldots 3.0$ were obtained.

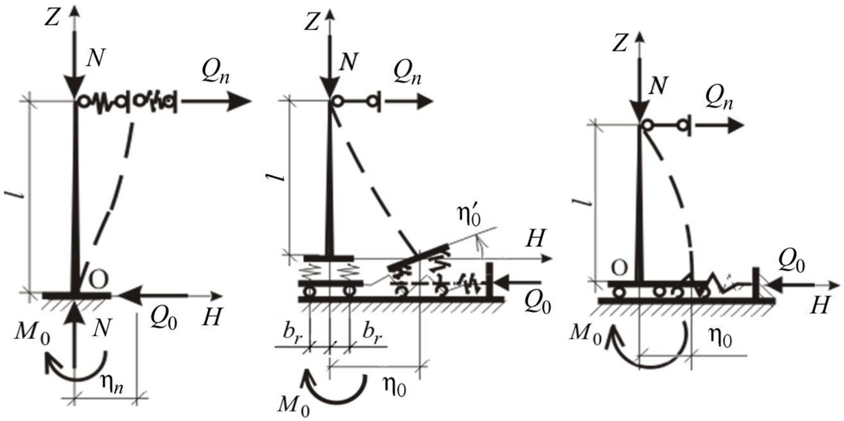

Fig. 3. Cases of resistance centrally compressed rods with variable cross section on elastic supports: (a) - case 3; (b) – case 4; (c) - case 5

Table 3
Effective length factor for variable cross-section column with fixed-support and elastic articulated support for b_{E3} =0 … 3.0 (case 3)

I_{xn}/I_{x0}	μ_x					
	b_{E3} =0	b_{E3} =0,05	b_{E3} =0,1	b_{E3} =0,5	b_{E3} =1	b_{E3} =3
0,99	0,70091	0,80758	0,99647	1,55882	1,73845	1,901
0,90	0,71772	0,81906	1,00456	1,57401	1,75817	1,926
0,80	0,73886	0,83412	1,01493	1,59273	1,78250	1,957
0,70	0,76330	0,85235	1,02724	1,61390	1,81005	1,992
0,60	0,79213	0,87486	1,04226	1,63827	1,84178	2,032
0,50	0,82708	0,90340	1,06130	1,66702	1,87921	2,080
0,40	0,87112	0,94093	1,08673	1,702122	1,92485	2,1382

Case 4 (Fig. 3(b)). Column has the mobile elastic fixed support with horizontal springy support for cross section with maximum dimensions (z=0) and articulated support for cross section with minimum dimensions ($z=l$). η_0 – displacement cross section with coordinate z_0=0, displacement of articulated elastic support. k_r – the coefficient of rigidity of elastic fixed-base. Coefficient of rigidity of horizontal springy support is k_{r2}. Boundary conditions are:

$$Q_0 = Q_n = +k_{r2}\eta_0,\ \ \eta_n = 0,\ \ M_{xn} = 0, \tag{13}$$

$$M_{x0} = -0,5k_r\eta_0' b_r^2,\ \ -\eta_0 N + M_{x0} + Q_0 l = 0. \tag{14}$$

Recurrent formula for the relationship between the angle of rotation mobile elastic fixed support and deflection of horizontal springy support is:

$$-\eta_0 N + M_{x0} + Q_0 l = 0,\ \ \eta_0 = \frac{0,5k_r b_r^2}{l \cdot k_{r2} - N} \cdot \eta_0'. \tag{15}$$

Substitute in equation (16) deflection of horizontal spring support on angle of rotation for mobile elastic fixed support by formula (19) leads to stability loss criterion of frame.

$$\left(\frac{k^2(EI_{x0}/l)}{0{,}5b_r^2k_r}+\frac{1}{1-k^2\,EI_{x0}\big/(k_{r2}l^3)}-\frac{\gamma_y}{2}\right)\cdot\frac{\mathrm{tg}(vu_n)}{v\gamma_y}+1=0\,. \qquad (16)$$

If $k_{r2}\to\infty$; than boundary condition have form (9) and stability loss criterion (16) converted to stability loss equation of column (10) – case 2.

If $k_{r2}\to 0$; than stability loss criterion (16) converted to stability loss equation of column (7), – case 1.

Case 5 (Fig. 3(c)). Column has the mobile fixed support with horizontal spring support for cross section with maximum dimensions (z=0) and articulated support for cross section with minimum dimensions ($z=l$). η_0 – displacement cross section with coordinate z_0=0, displacement of articulated elastic support. Coefficient of rigidity of horizontal springy support is k_{r2}.

If in equation (16) put the condition $k_r\to\infty$; it gives stability loss criterion of column for case 5:

$$\left(\frac{1}{1-k^2EI_{x0}\,/\,k_{r2}l^3)}-\frac{\gamma_y}{2}\right)\cdot\frac{\mathrm{tg}(vu_n)}{v\gamma_y}+1=0. \qquad (17)$$

If in equation (17) put the condition $k_{r2}\to\infty$; than stability loss criterion column with fixed- support and free-end (7).

Present methodology obtain coefficient the effective length of portal frame to calculate the critical buckling load of column with varying cross-section.

Portal frame has columns with varying cross-section and rigid frame rafter with constant cross-section. Columns with varying cross-section of portal frame is elements, which have boundary condition: column has the mobile elastic fixed support and articulated support for cross section with minimum dimensions, k_r - the coefficient of rigidity of elastic fixed-base. Coefficient of rigidity of horizontal springy support is k_{r2}=0.

Critical buckling load on the column with varying cross-section of portal frame may be calculate by using equation (7) or (16) for $k_{r2}\to\infty$.

Critical buckling load of frame depends from factor of elastic fixed-support, the ratio of angle of rotation node joint rafter and column corresponding of bending which is acting in the node:

$$b_{Er}=\frac{EI_{x0st}}{h_{st}}\frac{\eta'_{0k}}{M_{xk}}=\frac{EI_{x0st}}{EI_{x0r}}\frac{l_r}{h_{st}}M_\eta\psi_r,.\;\;\psi_r=\int_0^l\frac{(1-z\,/\,l)^2}{(1-\gamma_y z\,/\,l)^2}dz, \qquad (18)$$

where: M_{xk} – bending moment in node, which is acting due to loss stability of frame of asymmetric shape; η'_{0k} – the angle of rotation node joint rafter and column, is acting from loss stability steel frame of asymmetric shape; h_{st} – length of column of frame; E modulus of elasticity of steel; I_{x0st} – maximum moment of inertia of variable cross-section of column.

The coefficient ψ_r is the integral parameter of the stiffening cornice units of the jamming of the column in the cornice units, and determined by integrating of (18,19), takes into account the cross section of the rafter in determining the cornice unit rotation angle.

$$\psi_r = l\frac{1}{\gamma_y^3}\{-(\gamma_y - 1) - 2(\gamma_y - 1)\ln(1-\gamma_y) + \gamma_y\} - l\frac{1}{\gamma_y^3}(\gamma_y - 1)^2 .. \quad (19)$$

Performed numerical study of effective length factor of columns with varying cross-section of portal frame due loss stability of frame of asymmetric shape Table 4.

Table 4

Effective length factor of column of portal frame (EI_{x0st}/EI_{x0r} =1,0; length of column H=4…8 m), case 1

$\frac{I_{xnst}}{I_{x0st}} = \frac{I_{rxn}}{I_{rx0}}$	$EI_{x0st}l_r/EI_{x0r}h_{st}$		
	1,0	2,0	3,0
0,999	2,6348	3,18	3,652
0,9	2,671	3,2219	3,7
0,8	2,712	3,27	3,755
0,7	2,759	3,323	3,816
0,6	2,812	3,387	3,8865
0,5	2,876	3,46	3,969
0,4	2,952	3,54	4,07
0,3	3,05	3,61	4,198
0,25	3,112	3,732	4,277
0,2	3,187	3,817	4,373
0,1	3,4135	4,073	4,66

It is also proposed to review the work frame structures with two-hinged frames of I-section with respect to the passage length to the height of racks of 1: 3 in terms of the spatial loss of stability in the composition of buildings and structures. The horizontal elements of bond systems should be computed not only for external transverse loads, but for additional efforts that occur in the compressed-bent frame widths[61]:

$$S_e = S_0 + S_{\text{fic}} . \quad (20)$$

where: S_0 – efforts by external loads, S_{fic} – additional efforts at buckling frame of structures.

To find the total value of the load on the brace system it is necessary to calculate the value of the longitudinal force at unfastening. Since the materials and methods for determining the factors influence the structure of the whole structure for the stability of individual diameters practically no numerical studies have been conducted in the software package ANSYS by using calculation modules «Static Structural» and «Linear Buckling». Module «Linear Buckling» allows finding the critical value of the load in the design and getting graphical chart deformations in various forms of stability loss.

To obtain data on the critical load values (P_{buckling}) 11 models with different number of frames and brace systems of structures have been built and designed. Computational models can be divided into the following main groups:

- Frames without brace system;
- Frames with brace systems on the building ends.

This distribution calculation models performed are aimed at determining the effect of different types of brace systems or their absence on individual frames and building stability as a whole.

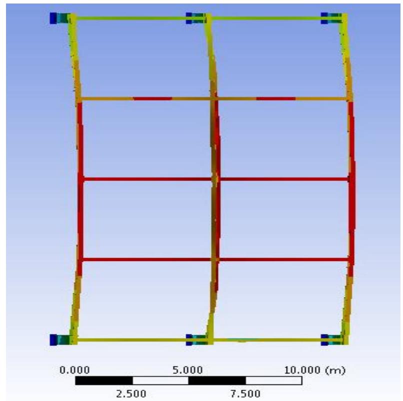

Fig. 4. Buckling shape of the frames in the structure without brace system

Let us consider the calculation of the results and analyze deformed buckling diagram of the building frame shown in Figure 4 and Figure 5.

Without brace system in the structure, buckling of unit frame leads to deformations of other frames, namely:

- deformation of the elements of restraint;
- buckling of other major load-bearing elements of the building structure.

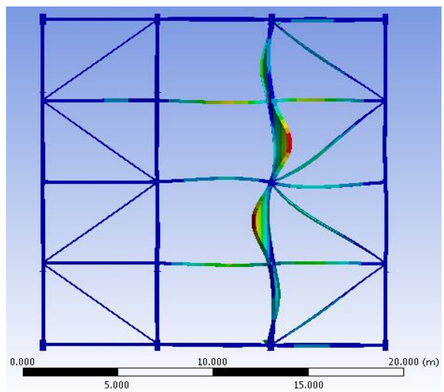

Fig. 5. Buckling shape of the frames in the structure with brace system on the ends of building

For the structures with bracing system at the ends, shape of buckling of a single frame is different: the buckling of the frame occurs by one half-wave sine wave between points of restraint from the plane. The value of the critical load on the system is increased by 25% in comparison with a similar system without constraints.

These results demonstrate the necessity of the calculation of brace system not only for the action of the external loads effect, but for the perception of additional lateral forces [S_{fic}] to provide the necessary rigidity and overall stability of the entire building.

To facilitate iterations in the software package, the critical load is calculated according to the equation [62]:

$$P_{\text{buckling}} = P_{\text{actual}} \cdot \lambda, \tag{28}$$

P_{buckling} – critical load; P_{actual} – actual load; λ – load factor (load multiplayer).

Figure 6 shows depending λ of the number of frames for structures without brace systems, and with brace systems at the ends of the building.

Figure 6 clearly demonstrates the impact spatial stiffness of building structure on the critical load for a particular frame. The difference between the critical load values for different layout options ranges from 16% (for structures with 6 frames) to 67% (for structures with two frames).

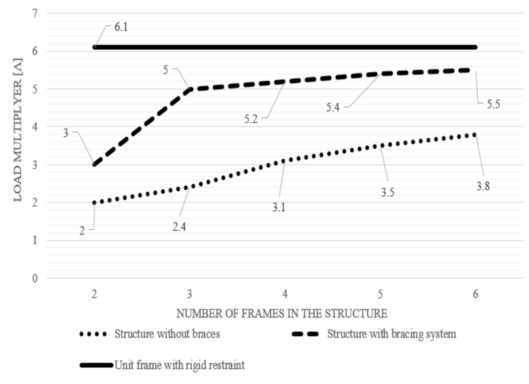

Fig. 6. Graph of the relationship between the critical load factor and number of frames in the longitudinal direction of the building

Conclusions. Buckling of a portal frame is very important problem of design steel constructions, composed of members with variable cross-section.

In this paper, study the elastic stability of the column with of variable cross-section.

For portal frames is presented methodology to determine critical load and effective length factor for buckling of a frame with tapered and with prismatic columns.

The buckling of columns with varying cross-section of portal frames results in the stability loss of frame asymmetric shape or symmetric shape.

The buckling columns with varying cross-section of portal frame causes the necessity to consider cases of buckling columns with the mobile elastic fixed support with horizontal springy support for cross-section with maximum dimensions z=0, and articulated support for cross-section with minimum dimensions, z=l.

Critical buckling load of frame depends on the factor of elastic fixed-support that in its turn depends on the ratio of joint rafter and column node rotation angle corresponding to bending acting in the node.

This simple method in the first design of steel frame makes it possible to obtain effective length factor of column variable cross-section of portal frame.

The formulation of the problem is based on the exact solution of the governing equations for buckling.

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СТІЙКІСТЬ СТАЛЕВИХ РАМ ІЗ ДВОТАВРІВ ІЗ ЗМІННОЮ ВИСОТОЮ СТІНКИ

Представлено дослідження стійкості сталевих портальних рам з двотаврів зі змінною висотою стінки. Залежно від жорсткості вузлів і умов закріплення елементів в рамах досліджено стійкість колон. Пропонується підхід для визначення стійкості рам через стійкість колон на пружних опорах. Жорсткість вузлів і пружність опор визначається із статичного розрахунку рами. Розглянуто п'ять різних випадків стійкості пружних стрижнів при різних крайових умов закріплення колон на пружних опорах. Проведено чисельні дослідження коефіцієнтів розрахункової довжини елементів портальних рам при змінності перерізу і жорсткості колон. Розвинений підхід до визначення стійкості елементів рам з площини рам залежно від піддатливості системи в'язів, проведений аналіз стійкість будівлі з комп'ютерним моделюванням і з обчисленням коефіцієнтів стійкості, фактору впливу системи в'язів рами на стійкість ригелів з площини портальних рам.

Ключові слова: змінне поперечний переріз; пружні опори; ефективна розрахункова довжина, коефіцієнт розрахункової довжини колон рам, вигин рами, фактор навантаження.

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УСТОЙЧИВОСТЬ СТАЛЬНЫХ РАМ С ДВУТАВРОВ С ПЕРЕМЕННОЙ ВЫСОТОЙ СТЕНКИ.

Представлено исследование устойчивости стальных портальных рам из двутавров с переменной высотой стенки. В зависимости от жесткости узлов и условий закрепления элементов в рамах исследована устойчивость колонн. Предлагается подход для определения устойчивости рам через устойчивость колонн на упругих опорах. Жесткость узлов и упругость опор определяется из статического расчета рамы. Рассмотрено пять различных случаев устойчивости упругих стержней при различных краевых условий закрепления колон на упругих опорах. Приведены численные исследования коэффициентов расчетной длины элементов портальных рам при переменности сечения и жесткости колонн. Развит подход к определению устойчивости элементов рам из плоскости рам в зависимости от податливости системы связей, проведен анализ устойчивость здания с компьютерным моделированием и с вычислением коэффициентов устойчивости, фактора влияния системы связей рамы на устойчивость ригелей из плоскости портальных рам.

Ключевые слова: переменное поперечное сечение; упругие опоры; эффективная расчетная длина, коэффициент расчетной длины колон рам, изгиб рамы, фактор нагрузки.

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Білик С.І., Білик А.С., Нілова Т.О., Шпинда В.З., Цюпин Є.І. **Стійкість сталевих рам із двотаврів із змінною висотою стінки** // Опір матеріалів і теорія споруд: наук.-тех. збірн. – К.: КНУБА, 2018. – Вип. 100. – С. 140-154.

Приведена методика і проведено чисельні дослідження коефіцієнтів розрахункової довжини елементів портальних рам при змінності перерізу і жорсткості колон. Показано визначення фактора стійкості елементів рам з їх площини в залежності від піддатливості системи в'язів.
Табл. 4. Іл.6. Бібліогр. 62 назв.

UDC 624.012.3
Bilyk S.I., Bilyk A.S., Nilova T.O., Shpynda V.Z., Tsyupyn E.I. **Buckling of the steel frames with the I-shaped cross-section columns of variable web height //** Strength of Materials and Theory of Structures: Scientific-and-technical collected articles – Kyiv: KNUBA, 2018. – Issue 100. – P. 140-154.

Coefficients of the effective length factor of the welded variable I cross-section columns have been obtained.
Table 4. Fig. 6. Ref. 62.

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Билык С.І., Билык А.С., Нилова Т.А., Шпинда В.З., Цюпин Е.І. **Устойчивость стальных рам с двутавров с переменной высотой стенки.** // Сопротивление материалов и теория сооружений: науч.-тех. сборн. – К.: КНУСА, 2018. - Вып. 100. - С. 140-154.

Приведена методика и численные исследования коэффициентов расчетной длины элементов портальных рам при переменности сечения и жесткости колонн. Описан подход к определению устойчивости элементов рам из плоскости рам в зависимости от податливости системы связей.
Табл. 4. Рис. 6. Библиогр. 62 назв.

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