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## EFFECT OF STATIC LOADS ON THE NATURAL VIBRATIONS OF RIBBED SHELLS

O.P. Krivenko,

Candidate of Science (Engineering), Senior Researcher, Leading Researcher

*Kyiv National University of Construction and Architecture, Kyiv  
Povitroflotsky ave., 31, Kyiv, 03680*

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**Abstract.** The article is devoted to a further analysis of the natural vibrations of inhomogeneous shells under the action of static loads. The method of investigation is based on a unified methodology that combines the problems of static stability and the vibrations of elastic shells. The problems of natural vibrations take into account the presence of a prestressed state of the shell structure from the action of static loads. The presence of a static load significantly affects the spectrum of the natural frequencies of the shell. This approach allows us to determine the critical load by the dynamic criterion.

The method of investigating of inhomogeneous shells is based on the uniform methodological positions of the 3-d geometrically nonlinear theory of thermoelasticity and the finite-element method in the form of the moment finite-element scheme. So, a thin shell is considered by this method as a three-dimensional body which is modeled throughout the thickness by one isoparametric solid finite element with multilinear shape functions.

Two nonclassical hypotheses are used to describe the stress-strain state of a thin inhomogeneous shell. The kinematic hypothesis of deformed straight line in the thickness direction: though stretched or shortened during deformation, a straight segment along the thickness remains straight. This segment is not necessarily normal to the mid-surface of the shell. The displacements are assumed distributed linearly along the thickness, which is conventional in the theory of thin shells. The static hypothesis compressive assumes that the stresses in the fibers are constant throughout the thickness of the shell.

Modal analysis of a shallow ribbed panel demonstrates the effectiveness of the developed method. The natural frequencies and mode shapes are determined at each increment of static loading.

**Keywords:** thin elastic shell, rib, buckling, natural vibrations, static load.

**Introduction.** Thin-walled shell structures are widely used in many branches of modern engineering. Problems of stability and natural vibrations of shallow panels are classical in the theory of thin elastic shells. Numerous literatures are devoted to their study [1-14]. Methods and algorithms for solving nonlinear stability problems and determining the parameters of natural vibrations are investigated on this type of shells, mainly of constant thickness. The shells are designed step-variable thickness (reinforced with ribs and overlays) to increase the overall rigidity of the thin-walled structure (and, correspondingly, its load-bearing capacity) [1-4, 10-12]. Static loads significantly affect both the stress-strain state of the structure and its dynamic characteristics, which include frequencies and forms of natural vibrations [5-8, 15, 16]. But the investigation of the static load effect on the vibrations of shells even of constant thickness is a difficult and insufficiently studied problem of structural mechanics. In addition the determination of natural frequencies and forms of small vibrations of the structure is a necessary element of investigation the behavior of shells.

**1 Statement of the problem and method of investigation.** The purpose of the article is to use the method developed in [5] for calculating the natural frequencies and forms of the mode of thin rim shells, taking into account the action of the static load. The approach developed in [5] allows us to investigate comprehensively

geometrically nonlinear deformation and buckling of thin and medium thickness elastic shells of an inhomogeneous structure [1-4] and to analyze small vibrations of shells with respect to a reference deformed state caused by a static load taking into account large displacements and pre-stressed states.

The finite-element method [1-4] is constructed on the basis of geometrically nonlinear relations of the three-dimensional theory of thermoelasticity, the positions of the moment finite-element scheme (MFES) and the application of the universal spatial finite element (FE) to modelling thin shells. We approximate a thin shell by one spatial FE throughout the thickness to develop a finite-element shell model. This approach is effective for solving problems of nonlinear deformation, buckling, and natural vibrations of thin shells [1-5, 9, 17-22]. The structural elements of an inhomogeneous shell require that the FE be universal: it should be eccentrically arranged relative to the mid-surfaces of the casing, it should be possible to vary the thickness of the lateral edges of the FE; the lateral edges of the neighboring FEs should be in continuous contact; and it should be possible to model sharp bends in the shell.

The problem of nonlinear deformation, buckling, and postbuckling behavior of inhomogeneous shells is solved by a combined algorithm that employs the parameter continuation method, a modified Newton–Kantorovich method, and a procedure for automatic correction of algorithm parameters [1, 2]. The justification of the reliability of the obtained calculation results is based on a study of the convergence of finite-element solutions and their comparison with known analytical and numerical results.

The algorithm for investigating the natural vibrations of shells with an inhomogeneous structure [5] is realized by the subspace iteration method [23, 24].

Thus, the proposed integrated approach allows us to investigate the joint effect of the parameters of pre-stressed state and the shells' geometric characteristics on the buckling and natural vibrations of shells with complex structure. The pre-stresses significantly affect the spectrum of natural frequencies. Their allowance makes it possible to determine the bifurcation points and the value of the critical force by the dynamic criterion [25]. According to the dynamic criterion, the load which corresponds to the appearance of a negative value of the fundamental frequency is taken as critical.

## 2 Analysis of stability and natural vibrations of the ribbed panel.

Investigation of the stability of rib-reinforced shells is presented using the example of shallow spherical panel of square planform ( $K=2a^2/(Rh)=32$ ,  $a=60h$ ,  $R=225h$ ) with hinged edges, and subject to uniform pressure. The shell is reinforced from inside by two central cross-ribs (height  $h_r=3h$  and width  $b_r=2h$ ) (Fig. 1). The input data:  $h=1$  cm,  $E=2.1\cdot 10^6$  kg/cm<sup>2</sup>,  $\nu=0.3$ ,  $\rho=7.85\cdot 10^{-3}$  kg/cm<sup>3</sup>. Results are presented in terms of dimensionless parameters:  $\bar{q}=a^4q/(Eh^4)$ ,  $\bar{u}^1=u^1/h$ , where  $u^1$  is the deflection of the panel along the axis  $x^1$ .

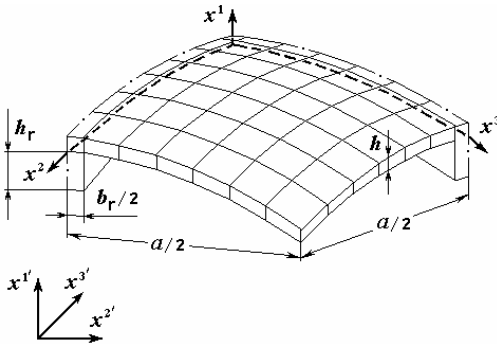


Fig. 1. Fragment of ribbed panel

Analysis for convergence shows that the  $16 \times 16$  FE mesh for whole panel is sufficient. To obtain the results with greater accuracy, the  $42 \times 42$  FE mesh has been used in the calculations.

The comparison of solutions is performed with data obtained for a smooth panel to analyze the effect of ribs on natural vibrations and the buckling of a shallow shell.

The results of the investigation of nonlinear deformation and the buckling of a smooth panel are given in Ref. [1, 2]. The results of the investigation of the natural vibrations for this panel are given in Ref. [5].

There is the dashed line with the mark “■” for the solution of the smooth panel on the “load – deflection” (“ $\bar{q} - \bar{u}$ ”) and “load – frequency” (“ $\bar{q} - \omega$ ”) curves. The results of calculations for the ribbed panel are marked “■”.

The circles on the “ $\bar{q} - \bar{u}$ ” and “ $\bar{q} - \omega$ ” curves indicate the loads  $\bar{q}^i$  for which the natural vibrations are calculated. The load  $\bar{q}^{i=0}$  corresponds to the initial state when the panel is not loaded ( $\bar{q} = 0$ ).

Analysis of the results (Table 1) shows us the following.

Under loads up to the value  $\bar{q}^{i=6}$  ( $\bar{q}^{i=0} \div \bar{q}^{i=5}$ ) the eigenfrequencies “ $\omega_2$ ” and “ $\omega_3$ ” are multiple. As a result the “ $\bar{q} - \omega_2$ ” and “ $\bar{q} - \omega_3$ ” curves coincide on this section (Fig. 2), and the corresponding mode shapes have the form shown in Fig. 3 ( $\bar{q}^{i=0}$ ).

Table 1

Eigenfrequencies of the ribbed panel for different values of the load  $\bar{q}^i$

No ( $\bar{q}^i$ )	$\omega_1, \text{Hz}$	$\omega_2, \text{Hz}$	$\omega_3, \text{Hz}$	$\omega_4, \text{Hz}$	$\omega_5, \text{Hz}$	$\omega_6, \text{Hz}$
0	493,16	761,98	761,98	793,29	970,44	1122,1
1	491,03	757,13	757,13	786,81	963,56	1116,3
2	484,35	746,97	746,97	773,92	950,21	1106,8
3	473,94	730,92	730,92	753,68	929,28	1092,0
4	457,36	704,72	704,72	720,93	895,60	1068,6
5	429,74	659,24	659,24	664,82	838,49	1030,2
6	378,78	553,51	567,29	567,29	727,84	960,48
7	321,23	482,05	502,72	502,72	664,92	934,08
8	287,05	402,90	436,65	436,65	593,95	895,02
9	224,04	241,95	309,07	309,07	470,04	833,85
10	106,79	145,85	224,59	224,59	413,22	807,73
11	- 982,64	101,45	159,47	159,47	373,40	787,21

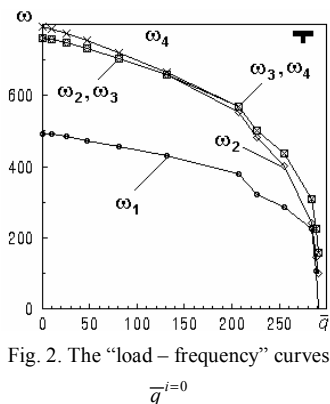


Fig. 2. The "load - frequency" curves

To the moment of buckling, the mode shape has an asymmetric form with maximum amplitude at the centers of the quarters (Fig. 3,  $\bar{q}^{i=10}$ ).

Starting from the load  $\bar{q}^{i=6}$ , we have a coincidence of the eigenfrequencies  $\omega_3$  and  $\omega_4$  (Table 1), the " $\bar{q} - \omega_2$ " and " $\bar{q} - \omega_3$ " curves (Fig. 2) and the corresponding reconstruction of the vibration modes (Fig. 3,  $\bar{q}^{i=6}$ ,  $\bar{q}^{i=10}$ ).

Throughout the range of load changing (from  $\bar{q}^{i=0}$  and until  $\bar{q}^{i=9}$ ), the mode shape that corresponds to the frequency  $\omega_1$  has a simple form (Fig. 3;  $\bar{q}^{i=0}$ ,  $\bar{q}^{i=6}$ ). The vibration is characterized by deformation in the area of the center of the panel.

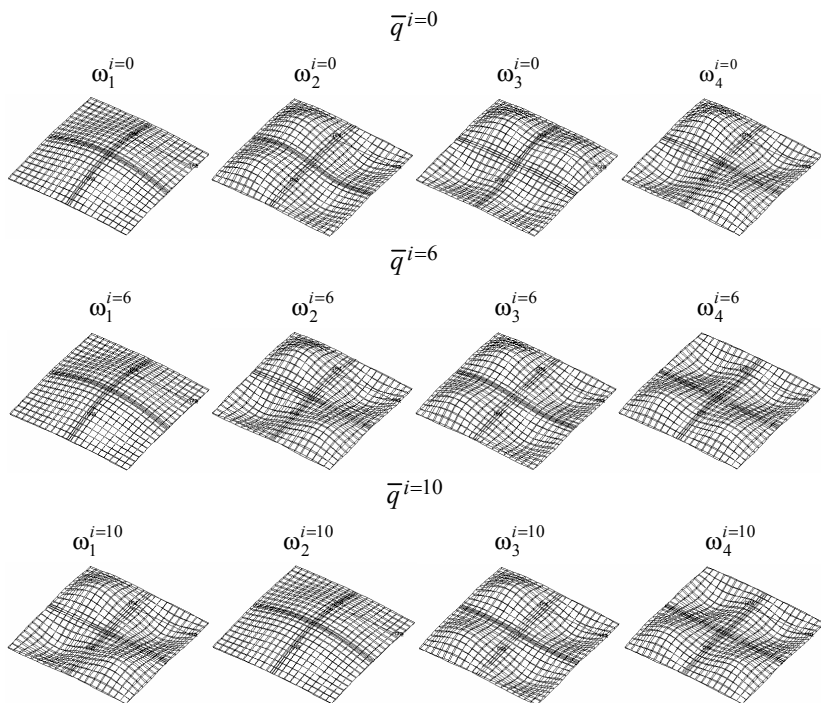


Fig. 3. Reconstruction of the first four mode shapes in the process of loading

A comparison of the "load-deflection" (Fig. 4) and "load-frequency" curves (Fig. 5) for smooth and ribbed panels has showed the following: adding two ribs increases the mass of the panel by 19.3% and  $\bar{q}_{cr}^{up}$  by a factor of 1.5 [1, 2]; and decreases the frequency  $\omega_1^{i=0}$  by 7.5% (for a smooth panel  $\omega_1^{i=0} = 533.78$ ). Upon

the load  $\bar{q} \approx 80 > \bar{q}^{i=4}$  the frequency  $\omega_1$  for the ribbed panel is greater than the corresponding frequency for a smooth shell.

According to the dynamic criterion when a frequency  $\omega_1 = 0$ , the load can be taken as critical. For a ribbed panel  $\bar{q}_{cr}^{up} = 291,4$ .

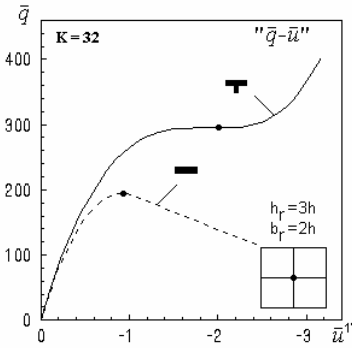


Fig. 4. The "load – deflection" curves"

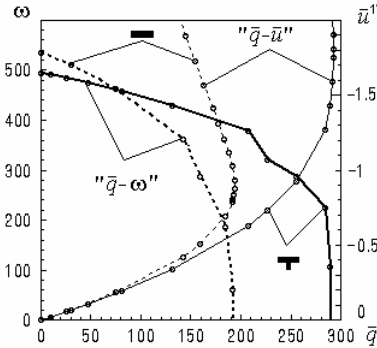


Fig. 5. Diagrams for smooth and ribbed panels

**Conclusions.** A method for studying the natural vibrations of inhomogeneous shells subject to action of static loads is considered. The method is based on a unified methodology that combines the problems of static stability and the vibrations of elastic shells. A modal analysis is performed on a pre-stressed structure. This approach allows us to determine the critical load by the dynamic criterion.

The natural frequencies and mode shapes are important parameters in the design of a thin-walled structure especially for dynamic loading conditions. These vibration characteristics of a structure may be served as a starting point for another, more detailed analysis.

The developed method has been used to investigate the buckling of a thin ribbed shell with the definition of frequencies and forms of natural oscillations at each increment of the load. Dependences of the vibration characteristics on the loading increase have been obtained.

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*Кривенко О.П.*

### **ВПЛИВ СТАТИЧНИХ НАВАНТАЖЕНЬ НА ВЛАСНІ КОЛИВАННЯ РЕБРИСТИХ ОБОЛОНОК**

Стаття присвячена подальшому аналізу власних коливань неоднорідних оболонок, що знаходяться під дією статичних навантажень. Метод дослідження заснований на єдиній методології, яка поєднує проблеми стійкості та коливань пружних оболонок. У задачі про власні коливання оболонки враховується наявність поперечно напруженого стану конструкції від дії статичних навантажень. Наявність статичного навантаження істотно впливає на спектр власних частот коливань оболонки. Такий підхід дозволяє визначити критичне навантаження за динамічним критерієм. Метод дає змогу розглядати широкий клас неоднорідних оболонок тонкої та середньої товщини. Під неоднорідністю оболонки

розуміться два види її особливостей: геометричні особливості конструкції у вигляді неперервно-змінної та ступінчасто-змінної товщини, зламів середньої поверхні, наявності отворів; структурна неоднорідність матеріалу оболонки за товщиною та у плані у вигляді комбінації різних багат шарових пакетів. Для опису напружено-деформованого стану тонкої неоднорідною оболонки використовуються дві гіпотези. Застосована некласична кінематична гіпотеза деформівної прямої: пряма в напрямку товщини, скорочуючись або подовжуючись, залишається прямою і після деформування оболонки. Ця пряма не обов'язково є нормальною до середньої поверхні оболонки. У напрямку товщини розподіл переміщень прийнято лінійним, що є загальноприйнятим підходом у теорії тонких оболонок. У статичній гіпотезі напруження обтиску волокон шару по товщині оболонки прийняті сталими.

Розроблений метод був використаний для дослідження стійкості тонкої ребристої оболонки з визначенням частот і форм власних коливань на кожному кроці приросту статичного навантаження. Отримано залежності характеристик коливань від збільшення навантаження. Модальний аналіз пологої ребристої панелі демонструє ефективність розробленого методу.

**Ключові слова:** тонка пружна оболонка, ребро, стійкість, власні коливання, статичне навантаження

*Кривенко О.П.*

### **ВЛИЯНИЕ СТАТИЧЕСКИХ НАГРУЗОК НА СОБСТВЕННЫЕ КОЛЕБАНИЯ РЕБРИСТЫХ ОБОЛОЧЕК**

Рассматривается методика исследования собственных колебаний тонкостенных оболочечных конструкций с учетом приложенной статической нагрузки. Методика опирается на комплексный подход, сочетающий конечно-элементный метод исследования геометрически нелинейного деформирования, устойчивости и закритического поведения тонких неоднородных оболочек и модальный анализ оболочек с учетом наличия напряженного состояния на каждом шаге нагружения. Выполнен анализ частот и форм собственных колебаний тонкой ребристой панели, шарнирно опертой по контуру и нагруженной равномерным давлением.

**Ключевые слова:** тонкая упругая оболочка, ребро, устойчивость, собственные колебания, статическая нагрузка

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*Наведено методику та результати розрахунку частот і форм власних коливань тонкої ребристою панелі, що шарнірно оперта по контуру та навантажена рівномірним тиском.*

Табл. 1. Іл. 5. Бібліогр. 25 назв.

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*The method for and results of calculating the frequencies and mode shapes of a thin ribbed panel hinged by the contour and loaded with uniform pressure are presented.*

Tabl. 1. Fig. 5. Bibliograf. 25 ref.

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*Приведена методика и результаты расчета частот и форм собственных колебаний тонкой ребристой панели, шарнирно опертой по контуру и нагруженной равномерным давлением.*

Табл. 1. Ил. 5. Библиогр. 25 назв.

**Автор (вчена ступень, вчене звання, посада):**

*кандидат технічних наук, старший науковий співробітник, провідний науковий співробітник НДІ будівельної механіки КНУБА КРИВЕНКО Ольга Петрівна*

**Адреса робоча:** 03680, Україна, м. Київ, Повітрофлотський проспект 31, Київський національний університет будівництва і архітектури

**Мобільний тел.:** +38(066) 048-32-77

**Робочий тел.:** +38(044) 245-48-29

**E-mail:** olakop@ukr.net

**ORCID ID:** <https://orcid.org/0000-0002-1623-9679>